Maths

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JUNIOR CERT MATHS PAPER ONE

An Introduction:
Mathematics is said to be "*the study of quantity, structure, space and shape*". Through its application to the simple and every day, as well as the complex and remote, it is true to say that mathematics is involved in almost all aspects of life and living. [Mathematics syllabus 2012]

Throughout this article material for Higher Level only will be clearly indicated.

Remember mathematics is not learned in isolation. It has significant connections with your other curricular subjects. Many elements of Science, Technical Graphics, Geography, Home Economics, Business Studies, Music and Art are all rich in mathematical structure.

Both Ordinary and Higher Level students have to attempt two papers.
(i) Each Ordinary Level paper will be of two hours duration.
(ii) Each Higher Level paper will be of two and a half hours duration.

There will be **six questions** on each paper and each question must be attempted. There will be **no choice**.

Each question will be broken into three parts:
Part (a) tests recall; very straight forward problem.
Part (b) is a more demanding question.
Part (c) tests application and is challenging.

Ordinary Level students will be given a **booklet** in which they will find the questions and work boxes for answers. Remember to show **all** your work clearly in these boxes.

Higher Level students will be given a question paper but their work must be presented on ordinary script paper that will be provided.

The symbol ✍ is very important. When it appears, it means that full marks will **not** be given if the answer on its own is presented. If the symbol ✍ is not given, then the answer on its own, if correct, will be sufficient. But as a general rule, always show your supporting work.

Paper One

An overview of topics covered in this paper for both Higher and Ordinary Levels:
* ARITHMETIC
* USE OF THE CALCULATOR
* ALGEBRA
* INDICES / SURDS
* SETS
* FUNCTIONS
* GRAPHS
* NUMBER SYSTEMS

Arithmetic

☆ Percentages

Familiarise yourself with a definite method for finding percentages.

Q. What is 15% of 300?
SOLUTION:
METHOD 1: Multiply 300 by $0 \cdot 15 = 45$

METHOD 2: Divide 300 by 100 and multiply by 15 = 45.

[ORDINARY / HIGHER]
Q. ✍ John bought an antique for €300, and sold it later for €500. Find his profit as a percentage of the cost price.

SOLUTION:

€500 – €300 = €200 = PROFIT

$$\frac{200}{300} \times \frac{100}{1} = \frac{20000}{300} = \frac{200}{3} = 66\tfrac{2}{3}\%$$

or use your calculator:

[200] [$a\frac{b}{c}$] [300] [×] [100] [=] [66r2r3]

☆ Ratio

[ORDINARY]
Q. ✍ Divide €350 in the ratio 3 : 4.
SOLUTION:
The question is asking you to divide €350 into 7 equal parts.

1 part ⇒ 350 ÷ 7 = 50
3 parts ⇒ 50 × 3 = 150
4 parts ⇒ 50 × 4 = 200
Answer: €150 and €200

[HIGHER]
Q. ✍ Mary and Bill divide a sum of money between them in the ratio 2 : 7 respectively. If Bill gets €60 more than Mary, find the sum of money.
SOLUTION:
Between Mary and Bill the sum of money is divided into 9 parts.
2 for Mary and 7 for Bill.
∴ €60 = 5 parts,
that is the difference between 2 and 7.

If 5 parts = €60
⇒ 1 part = €12
⇒ 9 parts = €12 × 9 = €108
Answer: €108

☆ Taxation

Make sure you understand the terms : gross salary (pay), gross tax, Standard Rate Cut – off Point (SRCP), tax credits, tax payable, net (take home) pay.

[ORDINARY / HIGHER]
Q. ✍ Tom earns €45000. He has a Standard Rate Cut-off Point of €15,000 and a tax credit of €7250. If the standard rate of tax is 20% and the higher rate is 42%, how much tax will he pay?
SOLUTION:
STEP 1: Gross Pay = €45,000
STEP 2: Gross Tax = 20% of SRCP + 42% of what remains
= 20% of 15000 + 42% of 30000
= €3000 + €12600
= €15600
STEP 3: Tax Payable
= Gross Tax - Tax Credits
= €15600 - €7250
= €8350
Answer: €8350

Note: ***Higher Level students*** need to practise a number of variations of the above problem.

☆ Currency Exchange

Remember the Rule:
Always keep the currency you are looking for on the **right**.

[ORDINARY / HIGHER]
Q. ✍ If €1 = $1.25 (dollars),
(i) change €175 to $ (dollars)
(ii) change $120 to € (Euro).
SOLUTION:
(i) We need dollars; keep them on the **right**.
€ 1 = $ 1.25
€ 175 = $ 1.25 × 175
= $ 218.75

(ii) We need Euro; keep them on the **right**.
$ 1.25 = € 1
$ 1 = € $\frac{1}{1 \cdot 25}$
$ 120 = € $\frac{1}{1 \cdot 25} \times 120$
= € 96

☆ Using the Calculator

Familiarise yourself with your calculator. It is a good idea to show each stage of a calculation when using a calculator. The two areas where mistakes are commonly made when using a calculator are **fractions** and **scientific notation (index notation)**.

☆ Fractions

Q. ✍ Calculate:
(i) $\frac{2+3}{5}$ (ii) $\frac{36+34}{30-23}$
(a) without using a calculator
(b) using a calculator.
SOLUTION:
(a) Without a calculator : Remember
(i) Do Top (ii) Do Bottom
(iii) Then Divide.

(i) $\frac{2+3}{5} = \frac{5}{5} = 1$

(ii) $\frac{36+34}{30-23} = \frac{70}{7} = 10$

(b) With a calculator : Remember that you must use brackets.

(i) $(2+3) \div 5 = 1$
(ii) $(36+34) \div (30-23) = 10$

☆ Scientific Notation (Index Notation)

A number written in the form $a \times 10^n$, where $1 \le a < 10$, $n \in Z$ is said to be written in Scientific Notation.

27000 in scientific notation is $2 \cdot 7 \times 10^4$.

154,000 in scientific notation is $1 \cdot 54 \times 10^5$.

Q. ✍ Evaluate $3 \cdot 7 \times 10^5 + 2 \cdot 8 \times 10^4$, and give your answer in scientific notation.
SOLUTION:
[3·7] [Exp] [5] [+] [2·8] [Exp] [4] [=]
$398000 = 3 \cdot 98 \times 10^5$

Note: Although the symbol ✍ is used before the question, you can still use your calculator provided you show the examiner in block form the keys used on the calculator.

Note: The key on the calculator we used is [Exp].

☆ Estimations

What is an estimate of each of the following:
(i) $\sqrt{18}$ (ii) $\sqrt{79}$?

(i) The nearest perfect square to 18 is 16 and $\sqrt{16}=4$. Thus an estimate of $\sqrt{18}$ would be 4.
(ii) The nearest perfect square to 79 is 81 and $\sqrt{81}=9$. Thus an estimate of $\sqrt{79}$ would be 9.

Q. ✍ By rounding to the nearest whole number, estimate the value of

$$\frac{1}{3\cdot 67}+(9\cdot 87)^2\times\sqrt{27\cdot 6}.$$

SOLUTION:

$$\frac{1}{4}+(10)^2\times\sqrt{25}$$
$$=0\cdot 25+100\times 5$$
$$=500\cdot 25$$

Algebra

Lots of basic algebraic operations and skills will be tested. Familiarise yourself with the following.

* SIMPLIFYING ALGEBRAIC EXPRESSIONS
* FACTORISATION
* DIVISION
* SOLVING EQUATIONS
* WORD PROBLEMS LEADING TO EQUATIONS

☆ Simplifying Algebraic Expressions

Q. ✍ Simplify

$$5(2p+q^2)+2p(3p+5q-5)$$
$$-5q(q+2p)$$

SOLUTION:

$$10p+5q^2+6p^2+10pq$$
$$-10p-5q^2-10pq$$
$$=6p^2$$

Remember: Simplify means "tidy up".

☆ Factorisation

1. COMMON FACTORS

Q. Factorise $8ab^2-16a^2b+24a^2b^2$
Answer: $8ab(b-2a+3ab)$

2. GROUPING

Q. Factorise $2ax+4ay-3bx-6by$
Answer: $(2a-3b)(x+2y)$

3. QUADRATICS

Q. Factorise $2x^2-x-10$
Answer: $(2x-5)(x+2)$

4. DIFFERENCE OF TWO SQUARES

Q. Factorise $16x^2y^2-81p^2$
Answer: $(4xy-9p)(4xy+9p)$

5. COMBINATION TYPE **[HIGHER]**

Q. Factorise $4x^2+12x+9-16y^2$
Answer: $(2x+3-4y)(2x+3+4y)$

☆ Division [Higher]

Q. ✍ Divide $6x^3+2x^2-29x+15$ by $3x-5$
Answer: $2x^2+4x-3$

☆ Solving Equations

Equations are asked in every exam so make sure you are familiar with the following types.

TYPE 1: Linear equations (no x^2)

Q. ✍ Solve $10-8(x-6)=x-4(x-2)$
Answer: $x=10$

TYPE 2: Quadratic equations (contain x^2)

Q1. ✍ Solve $2x^2-13x+18=0$
Answer: $x=4\cdot 5$ or $x=2$

Q2. [HIGHER] ✍ Solve $3x^2-x-3=0$, correct to two decimal places.
SOLUTION: *'Solve, correct to two decimal places' is a direct hint to use the formula.*

$$3x^2-x-3=0 \qquad a=3,\ b=-1,\ c=-3$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Answer: $x=1\cdot 18$ or $x=-0\cdot 85$

TYPE 3: Simultaneous Equations

Q. ✍ Solve $5x-2y-13=0$
$3x-4y-12=0$
Answer: $x=2,\ y=-1\cdot 5$

☆ Word problems leading to (i) a quadratic equation, or (ii) simultaneous equations

(i) **[HIGHER]** : Quadratic Equation Type

Q. ✍ A prize fund of €750 was divided among a group of winners. If there had been 5 more winners, each winner would have received €25 less. Find the original number of winners.

SOLUTION:
STEP 1: Let $x=$ original number of winners.
STEP 2: Make a grid.

	Original Prize	New Prize
Number of winners	x	$x+5$
Prize fund	€750	€750
Each prize	$\frac{750}{x}$	$\frac{750}{x+5}$

STEP 3: Re-read the question and form an equation.

New Prize = Original Prize − 25

$$\frac{750}{x+5}=\frac{750}{x}-25$$

STEP 4: Solve this equation for x.
Answer: $x=10$
(There were 10 original winners.)

(ii) **[ORDINARY / HIGHER]**
The Simultaneous Equation Type

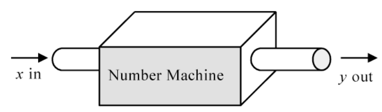
METHOD:
STEP 1: Let $x=$ one unknown number
Let $y=$ the other unknown number.
STEP 2: Look for 2 facts that link x and y and form two equations.
STEP 3: Then solve these simultaneous equations.

Q. ✍ A teenager has a total of 36 CDs, some of which are valued at €10 each and the rest are valued at €5 each. The total value of the CDs is €245. How many of each type of CDs has the teenager got?

SOLUTION:
STEP 1: Let $x=$ number of CDs valued at €10 each.
Let $y=$ number of CDs valued at €5 each.
STEP 2: Form two equations
(i) $x+y=36$
(ii) $10x+5y=245$
STEP 3: Now solve these simultaneous equations:
$x=13$ and $y=23$
Answer : The teenager has 13 CDs valued at €10 each and 23 CDs valued at €5 each.

Functions

A function is a RULE that changes one number, called an input, into another number, called an output. We could think of a function as a NUMBER MACHINE that changes an input x into an output $y=f(x)$.



A function will generally be represented in one of two ways.

(i) $f:x\to x^2$ or (ii) $f(x)=x^2$
(↑ in, ↑ out) (↑ in, ↑ out)

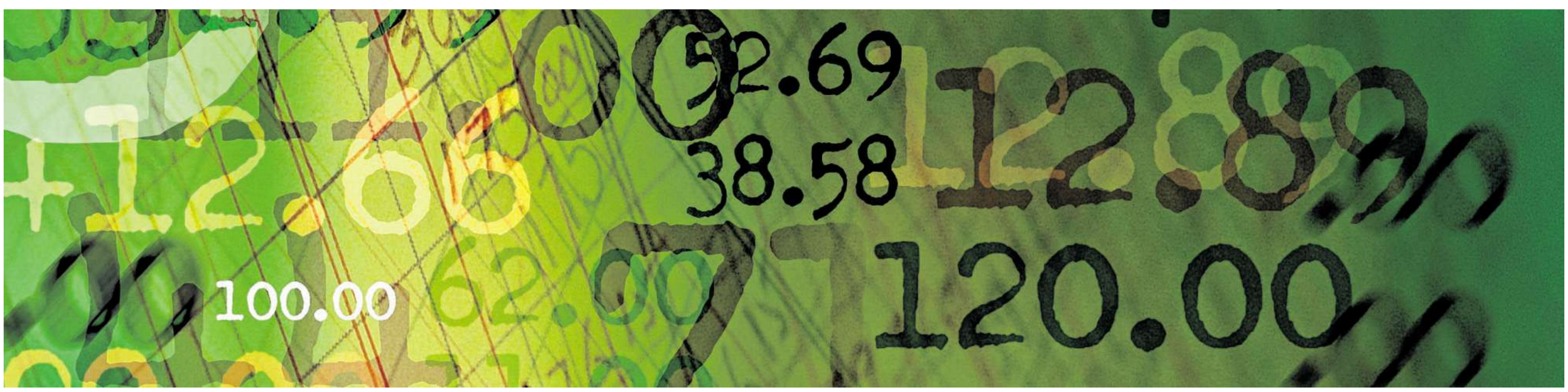
Consider the following five types of questions.

TYPE 1: Evaluate a Function

Q. ✍ If $f(x)=5x-3$, evaluate $f(2)$.
SOLUTION:
$f(x)=5x-3$
$f(2)=5(2)-3=7$ [2 in, 7 out]

TYPE 2: Find x when given the value of the function.

Q. ✍ If $f(x)=5x-2$, find x for which $f(x)=18$.
SOLUTION:
$f(x)=5x-2$
But $f(x)=18$
$\Rightarrow 5x-2=18$
$5x=20$
$x=4$ [18 out, 4 was put in]



TYPE 3: **[HIGHER]** Difficult Substitutions

Q. If $f(x)=x^2-4$, write an expression for $f(x-2)$.

SOLUTION:

$f(x)=x^2-4$

$f(x-2)=(x-2)^2-4$

$=x^2-4x+4-4$

$=x^2-4x$

[$x-2$ in, x^2-4x out]

TYPE 4: **[HIGHER]** Looking for a missing number.

Q. $f:x\to p-2x^2$ is a function, find p if $f(-2)=12$.

SOLUTION:

$f(x)=p-2x^2$

$f(-2)=p-2(-2)^2=12$

$\Rightarrow\ p-8=12$

$\Rightarrow\ p=20$

TYPE 5: **[HIGHER]** The Simultaneous Equations Type.

Q. $f(x)=x^2+px+q$ is a function. If $f(1)=2$ and $f(-1)=12$, find the value of p and the value of q.

SOLUTION:

STEP 1: $f(x)=x^2+px+q$

$f(1)=1^2+p(1)+q=2$

$\Rightarrow\ p+q=1$

STEP 2: $f(x)=x^2+px+q$

$f(-1)=(-1)^2+p(-1)+q=12$

$\Rightarrow\ -p+q=11$

STEP 3: Solve the simultaneous equations.

Answer: $p=-5$ and $q=6$

Graphs

Most students like drawing graphs. Make sure you know the difference between (i) a linear function and (ii) a quadratic function.

Note 1: The graph of a linear function (one with no squares) will be a straight line, e.g. $f:x\to 3x-7$.

Note 2: The graph of a quadratic function will be a smooth curve, e.g. $f:x\to x^2+x-6$.

Remember: For Ordinary Level students the graph paper is in the booklet. Higher Level students must ask the superintendent for graph paper.

Q. Draw the graph of the function $f:x\to 2x-1$ in the domain $-1\le x\le 3$.

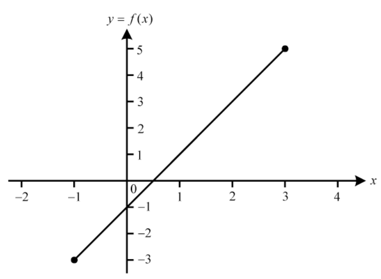
SOLUTION:

STEP 1: Make a table and take the endpoints of the domain as your two x values.

x	$2x-1$	$y=f(x)$
-1	$2(-1)-1$	-3
3	$2(3)-1$	5

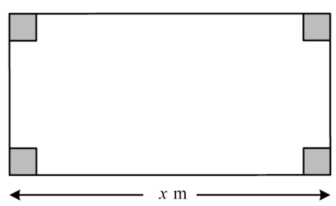
$\therefore$ one endpoint is $(-1,-3)$ and the other is $(3,5)$

STEP 2: The Graph



[HIGHER]

Q. A farmer has 12 metres of wire fencing and four posts with which to make a rectangular enclosure in the middle of a field for sheep. One side of the rectangle is x metres long.



(a) Show that the area enclosed will be $6x-x^2$ square metres.

(b) Draw the graph of the function $f:x\to 6x-x^2$ in the domain $0\le x\le 6$.

(c) Use your graph to estimate
   (i) the area of the enclosure when $x=1\cdot 5$ metres,
   (ii) the maximum possible area of the enclosure,
   (iii) the two values of x for which the area is $4\cdot 5\text{m}^2$.

SOLUTION:

(a) Length of enclosure $=x$ metres

Breadth of enclosure $=6-x$ metres

Area of enclosure $=$ Length $\times$ Breadth

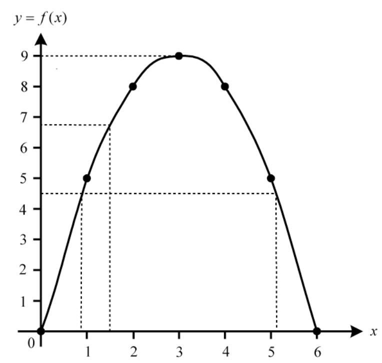
$=x(6-x)$

$=6x-x^2$ sq. metres

(b) STEP 1: Make a Table

x	0	1	2	3	4	5	6
$y=f(x)$	0	5	8	9	8	5	0

STEP 2: Draw the Graph



(c) **Note**: Answers must be worked out from the graph. Draw construction lines on the graph to show how you achieved your answer.

(i) Start at $x=1\cdot 5$ and draw a vertical line up until you meet the curve. Then draw a horizontal line over to the y-axis. **Answer** : $6\cdot 8\text{m}^2$

(ii) The maximum area is the highest y value. Go to the top of the graph and draw a horizontal line across to the y axis. **Answer** : 9m^2

(iii) Area is on the y-axis. Find $4\cdot 5$ on this axis. Draw a horizontal line across to cut the graph at two points. At these points, draw two vertical lines to meet the x-axis. Read off the x values at these points. **Answer** : $x=0\cdot 75$ or $5\cdot 1$.

Number Systems

Types of numbers and their symbols.

* The set **N** of natural numbers.
  N = {1, 2, 3, 4, 5......}.

Know: Place value, sets of divisors, pairs of factors, prime numbers, H.C.F, L.C.M, cardinal number, number line.

* The set **Z** of integers.
  Z = {...-3, -2, -1, 0, 1, 2, 3...}

*The set **Q** of rational numbers.
Decimals, fractions, percentages. Decimals and fractions plotted on the number line.

*[Higher only] The set **Ir** of irrational numbers. These are numbers that cannot be written as a fraction such as π $\sqrt{2}$, $\sqrt{3}$.

* The set **R** of real numbers. Every point on the number line represents a real number.

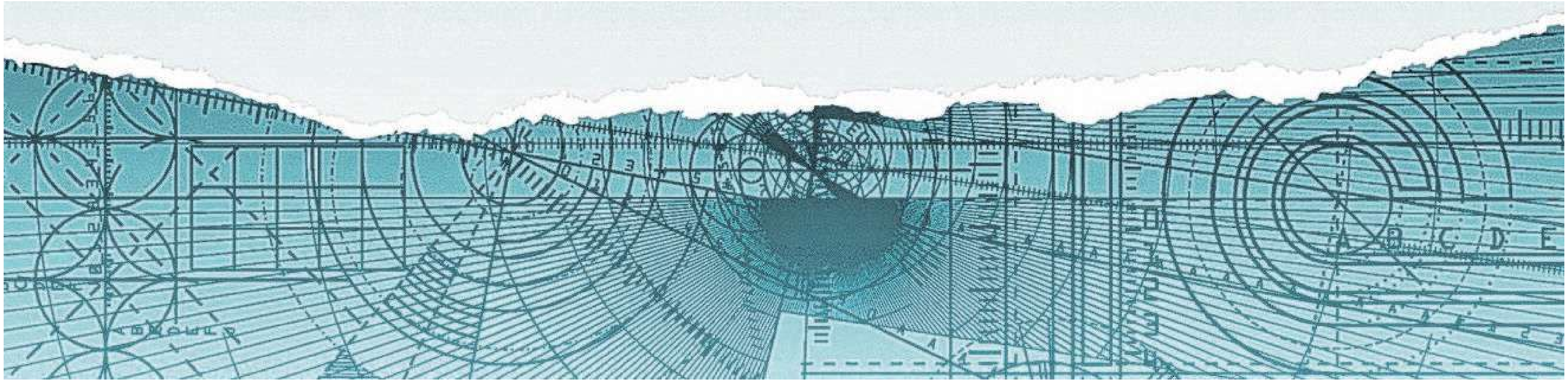
Know: Inequality Symbols (reading from left to right)

> "is greater than"

< "is less then"

≥ "is greater than or equal to"

≤ "is less than or equal to"



S PAPER TWO

Here is an overview of the topics covered in this paper for both ***Higher Level*** and ***Ordinary Level***.

- ☆ Perimeter and Area & Surface Area and Volume
- ☆ Project Maths: Phase 1
- ☆ Strand 1: Statistics and Probability
- ☆ Strand 2: Synthetic Geometry
  Transformation
  Trigonometry
  Co-ordinate Geometry

Area and Volume

Need to know:

[ORDINARY]

- ☆ **Perimeter and Area:** rectangle, square, triangle, parallelogram, circle (disc) and sector of a circle
- ☆ **Surface Area and Volume:** rectangular solid, cube, cylinder, sphere and hemisphere
- ☆ Find the **radius** of a container
- ☆ Find the **height** of a container (Depth of liquid)
- ☆ **Equal volumes**.

[HIGHER]

- ☆ **All of the above**
- ☆ **Cone**… where $r =$ radius; $h = \perp$ height; $l =$ slant height.
- ☆ **Combinations** of different shapes
- ☆ **Rates of Flow**

Note: Many of the formulae you require are on pages 8, 9, 10 and 11 of the **Maths Tables**. Table books are supplied in the exam; if you are not given one ask the supervisor for one.

[ORDINARY / HIGHER]

Q. ✍ Find the volume, in terms of π, of a cylinder with radius 5cm and height 7cm.

SOLUTION:

$$\text{Volume of cylinder} = \pi r^2 h$$
$$= \pi(5)^2(7)$$

Answer: $175\pi \text{ cm}^3$

Note: "Find in terms of π" means, leave π as π.

Q. ✍ A sphere has a volume of $36\pi \text{ cm}^3$. Find the radius of the sphere.

SOLUTION:

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

But volume $= 36\pi$

$$\frac{4}{3}\pi r^3 = 36\pi \quad (\text{cancel } \pi)$$
$$r^3 = \frac{3 \times 36}{4} = 27$$

Answer: $r = 3$ cm

[HIGHER]

Q. ✍ A cylinder of height 24cm and radius 3cm has the same volume as that of a cone of radius 6cm. Find the height of the cone.

SOLUTION:

STEP 1: $\text{Volume of cylinder} = \pi r^2 h$
$$= \pi(3)^2(24)$$
$$= 216\pi$$

STEP 2: $\text{Volume of cone} = \frac{1}{3}\pi r^2 h$
$$= \frac{1}{3}(\pi)(6)^2(h)$$
$$= 12\pi h$$

STEP 3: But volume of cone = volume of cylinder

$$\Rightarrow \quad 12\pi h = 216\pi \quad (\text{cancel } \pi)$$
$$\Rightarrow \quad 12h = 216$$

Answer: $h = 18$ cm

Note: Always leave π as π when comparing two volumes.

[HIGHER]

Q. ✍ Sweets, made from a chocolate mixture, are in the shape of solid spherical balls. The diameter of each sweet is 3cm. 25 sweets fit exactly in a rectangular box which has internal height 3cm. The base of the box is a square.

(i) How many sweets are there in each row?
(ii) What is the internal volume of the box?
(iii) Find the volume of air in the box when the 25 sweets are placed inside it. Give your answer correct to 2 places of decimals and use $\pi = 3 \cdot 14$.

SOLUTION:

(i) There are 5 sweets in each row.
(ii) Volume of box = L × B × H
= 15 × 15 × 3
= 675 cm³

(iii) Volume of air in box when 25 sweets are placed inside it
= volume of box – volume of 25 sweets

$$= 675 - 25\left(\frac{4}{3}\pi r^3\right)$$
$$= 675 - 353 \cdot 25$$
$$= 321 \cdot 75 \text{ cm}^3$$

Coordinate Geometry of the Line

Ordinary Level students should know how to use the formulae. These will be given on the paper. ***Higher Level*** students need to learn the formulae off-by-heart and know how to apply them. See page 18 of **Maths Tables**.

Questions asked in this topic can be grouped under the following headings.

- ☆ Problems involving **two points**
- ☆ Problems involving the **equation of a line**
- ☆ Problems involving **three points**
- ☆ Problems involving **two line equations**
- ☆ **Bits and Pieces**

☆ Two Points

Q. ✍ $a(-1,3)$ and $b(2,9)$ are two points

(a) Find $|ab|$.
(b) Find the midpoint of $[ab]$
(c) Find the slope of ab.
(d) Find the equation of ab.

Answers: (a) $\sqrt{45}$ (b) $\left(\frac{1}{2}, 6\right)$ (c) 2
(d) $2x - y + 5 = 0$

☆ The Equation of a Line

Q. ✍ K is the line $2x - y - 4 = 0$.

(i) Verify that $(4,4) \in$ K.
(ii) Find the slope of K.
(iii) Find where K intersects the x-axis.
(iv) Find where K intersects the y-axis.
(v) Find the image of K under a central symmetry in the origin.

Answers: (ii) 2 (iii) $(2,0)$ (iv) $(0,-4)$
(v) $2x - y + 4 = 0$

■ **Students at the Institute of Education, Lower Leeson St, Dublin 2.** PHOTOGRAPH: ALAN BETSON

☆ Three Points

Q. ✍ $a(4,2)$, $b(-2,0)$ and $c(0,4)$ are three points.
(i) Prove that $ac \perp bc$.
(ii) Prove that $|ac| = |bc|$.
(iii) Calculate the area of the triangle bac.

Answers: (i) Find the slope of ac. Call m_1
Find the slope of bc. Call m_2
Multiply m_1 by m_2
If ans = -1 $\Longrightarrow$ $ac \perp bc$.
(ii) Find the length of ac
Find the length of bc
Compare them $\Rightarrow |ac| = |bc|$
(iii) 10

☆ Two Line Equations

You need to know:
* Are they parallel or perpendicular ?
* How to find their point of intersection by (i) Graphing or (ii) Calculation

Q. ✍ K is the line $x - 2y - 3 = 0$ and L is the line $2x + y - 1 = 0$
(i) Investigate if $K \perp L$
(ii) By graphing lines K and L find their point of intersection.
(iii) By using simultaneous equations find the point $K \cap L$.

SOLUTION:
(i) Slope of $K = \frac{1}{2}$, slope of $L = -2$
As $\frac{1}{2} \times -2 = -1$, $K \perp L$.
(ii) Graph lines K and L on graph paper. Then from your graph read the point $K \cap L$ and you should get $(1,-1)$.
(iii) Solve the simultaneous equations.

K: $x - 2y = 3$
L×2: $4x + 2y = 2$
$5x \quad = 5$
$x = 1$
By substitution, $y = -1$
$\therefore$ $K \cap L = (1,-1)$.

☆ Bits and Pieces

Q1. ✍ The point $(k,1)$ lies on the line $4x - 3y + 15 = 0$. Find the value of k.
Answer: $k = -3$

Q2. ✍ The point $(-3,4)$ is on the line whose equation is $5x + y + k = 0$. Find the value of k.
Answer: $k = 11$

Q3. ✍ $a(1,6)$ and $b(-3,-1)$ are two points. The point $d = (2,y)$ is such that $|ab| = |ad|$. Find the two possible values of y.
Answer: $y = 14$ or $y = -2$

Trigonometry

Strand 2: Trigonometry

Right-Angled Triangles only
The Theorem of Pythagoras

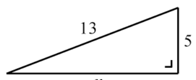
·**Ordinary level students** will be required to:
- Apply the result of the Theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving height and distances.
- Use trigonometric ratios, Sin, Cos and Tan to solve problems involving angles (integer values) between 0° and 90°.

Higher level students will be require to:
- know all of the above.
- Trigonometric ratio in Surd form for angles 30°, 45° and 60°.
- Solve problems involving surds.
- Manipulate measure of angle in both decimal and DMS forms.

[ORDINARY / HIGHER]

☆ Theorem of Pythagoras

Q. ✍ Find the length x correct to one decimal place.

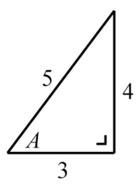


Answer: $x = 12$

☆ The Three Ratios : Sin, Cos and Tan

Memory Aid: **S**illy **O**ld **H**arry, **C**aught **A** **H**erring, **T**rawling **O**ff **A**merica

Q. ✍ Write down $\sin A$, $\cos A$ and $\tan A$.

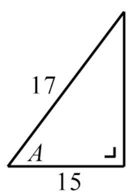


Answer:
$\sin A = \frac{4}{5}$
$\cos A = \frac{3}{5}$
$\tan A = \frac{4}{3}$

☆ The Three Ratios and Pythagoras

Q. ✍ If $\cos A = \frac{15}{17}$, find $\sin A$ and $\tan A$.

SOLUTION:
STEP 1: Draw a right-angled triangle.
STEP 2: Find the missing side, 8, using Pythagoras.



STEP 3: $\sin A = \frac{8}{17}$
$\tan A = \frac{8}{15}$

☆ The Calculator

Make sure you can calculate sin, cos and tan on your calculator. ***Ordinary Level*** students will **not** be asked to use minutes; ***Higher Level*** students could be.

Q. What is $\cos 37°42'$, correct to two places of decimals?

SOLUTION:
[cos] [37] [D°M′S] [42] [D°M′S] [=]

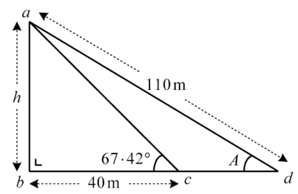
Answer: $0 \cdot 79$

Note: On some calculators, the [D°M′S] button is the [°′″] button.

☆ Solving Triangles

[HIGHER]

Q. ✍ A vertical pole ab stands on level ground. A straight wire joins a, the top of the pole, to c, a point on the ground. c is 40m from b, the bottom of the pole. A second straight wire joins a to d, another point on the ground. The length of this wire is 110m. $|\angle acb| = 67 \cdot 42°$.
Find
(i) $|ab|$, to the nearest metre,
(ii) $|\angle adb|$, correct to the nearest degree.



SOLUTION:
(i) In Δabc, let $|ab| = h$.

$$\tan 67 \cdot 42° = \frac{h}{40}$$

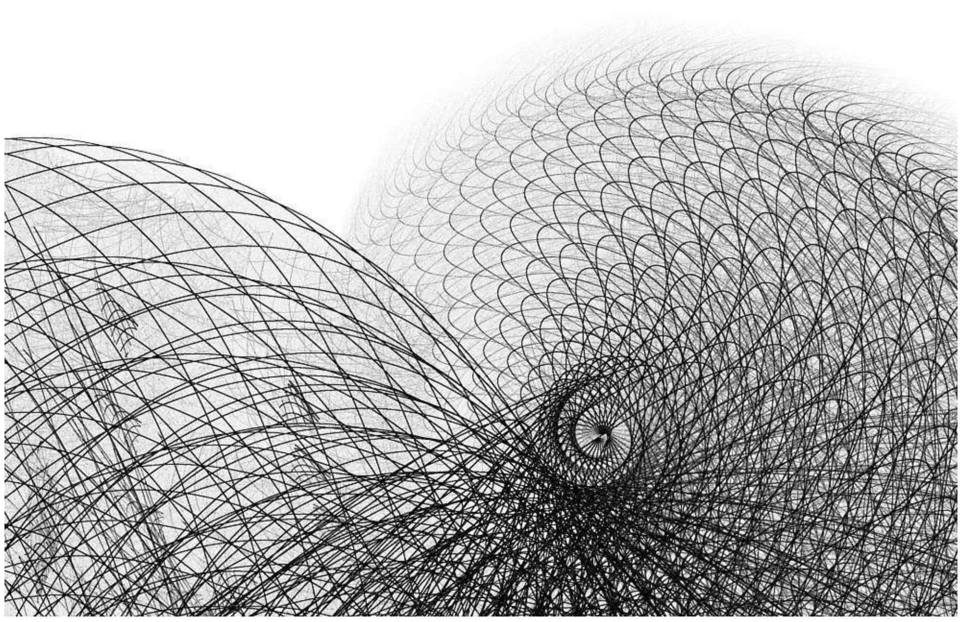
$$h = 96 \cdot 188 = 96\text{m}$$

(ii) In the Δabd, let $|\angle adb| = A$.

$$\sin A = \frac{96 \cdot 188}{110}$$

$$A = 60 \cdot 97° = 61°$$

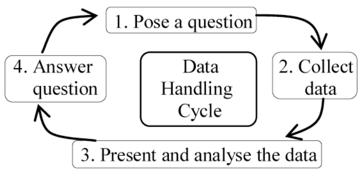
■ **Students at the Institute of Education, Lower Leeson St, Dublin 2.** PHOTOGRAPH: ALAN BETSON



Statistics

For most students the statistics question is a favourite. There are basically two sections to this topic.
* CALCULATE
* DRAW

Strand 1: Statistics
Work in this strand focuses on engaging you as learners in the process of data collection.



*Know the different data types.

1. Categorical Data – Nominal Data / Ordinal Data
2. Numerical Data – Discrete Data / Continuous Data

Q. [ORDINARY/HIGHER]

Indicate in the box provided whether each of the following is categorical or numerical data.
1. Goals scored in a football match.
2. Hair colour of your classmates.
3. Makes of cars in a car park.
4. Time take for athletes to run 100 metres.

Answer: 1. Numerical **2.** Categorical **3.** Categorical **4.** Numerical

Know:
1. Primary Data: This is first hand data, that is, data you collect yourself by means of a survey ·Personal interviews using a questionnaire ·Telephone surveys ·Observing what is happening ·Experiments. Very often it is not possible to collect data from everybody. In such cases, a sample is chosen and data is collected from the sample. This is known as a **sample survey**.

2. Secondary Data: This is second hand data that has already been collected. The Guinness Book of Records; The Census of Population; The Internet.

Know:
* Univariate Data
* Bivariate Data
* Sample
* A random sample
* Population
* Bias
* An outlier

☆ Representing Data

***Numerically**
Learn how to find Mean, Median, Mode and Range of a simple array (group) of numbers.
Q. 1 Find the mean, median, mode and range of 2, 3, 4, 10, 10, 10.
Answers: Mean=6.5, Median = 7, Mode = 10, Range = 8.

Q. 2 Find the mean and the mode of the following frequency table.

Ages of children	3	4	5	6	7
No.of children	4	6	3	10	2

Answers: Mean = 5, Mode = 6

[HIGHER]
Find the mean of a grouped frequency distribution. Remember to find the mid-interval value of each class.

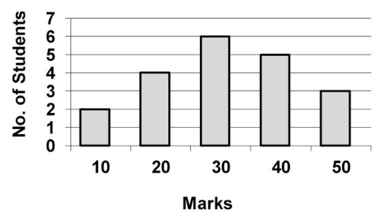
***Graphically**
To display data we use:
·Pie charts ·Bar charts ·Line plots
·Histograms ·Stem plots (stem and leaf diagram) You must always add a key to show how the stem and leaf combine.

Higher level students need the above and the following:
· Back-to-back stem and leaf plots to compare data sets
·Use stem plots to calculate quartiles and Interquartile range.
· Remember to add a key to each leaf to show how the stem and leafs combine.

Q. The table shows the marks gained in a test by 20 students.

Marks	10	20	30	40	50
No. of Students	2	4	6	5	3

Draw a bar chart of the data, putting marks on the horizontal axis.
Solution: The Bar Chart



Note: Higher Level Students:
For histograms with unequal class intervals the frequency ÷ by the base = the height.
You need to practise lots of these.

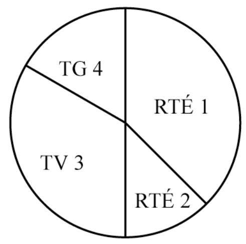
Q. A survey was taken of 96 students to find their favourite channel from four given TV channels. The table shows the results of the survey.

Favourite Channel	RTÉ1	RTÉ2	TV3	TG4
No.of students	36	12	32	16

Draw a Pie Chart of the data.

Solution:
Step 1: Convert the information into degrees.
(i) RTÉ1 = 36 ÷ 96 x 360 = 135°
(ii) RTÉ2 = 12 ÷ 96 x 360 = 45°
(iii) TV 3 = 32 ÷ 96 x 360 = 120°
(iv) TG 4 = 16 ÷ 96 x 360 = 60°

Step 2: Draw the Pie Chart.



Strand 1: Probability

Probability involves the study of the laws of chance. It is a measure of chance or the likelihood of something happening.
Learn the following terms:
1. A Trial: A trial is the act of doing an experiment. Each toss of a coin or throw of a die is a trial.

2. Outcome: The possible things that can happen from a trial are called outcomes. E.g. rolling a die: the outcomes are 1, 2, 3, 4, 5, or 6.

3. An Event: An event is the occurrence of one or more specific outcomes of an experiment. It is what you want to happen, e.g. getting an odd number from the roll of a die: answer = $\frac{3}{6} = \frac{1}{2}$

4. Sample Space: The set or list of all possible outcomes in a trial or an experiment.

5. The Fundamental Principle of Counting
If one event has **m** possible outcomes and a second event has **n** possible outcomes, then the two events have **m × n** possible outcomes.

6. For equally likely outcomes, the probability of event E occurring is given by
$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Numberof possible outcomes}}$$

Remember: The probability of an event, E, happening is a number between 0 and 1 including 0 and 1.
$$0 \leq P(E) \leq 1$$

7. The Probability Scale

0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Impossible	Unlikely	Even Chance	Likely	Certain

Q. An unbiased six sided die is thrown once. Find the probability that the number obtained is **(a)** 3 **(b)** Even **(c)** A Prime Number

Answers: (a) $\frac{1}{6}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{2}$

Q. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card will be
(a) A diamond **(b)** An Ace **(c)** An even number

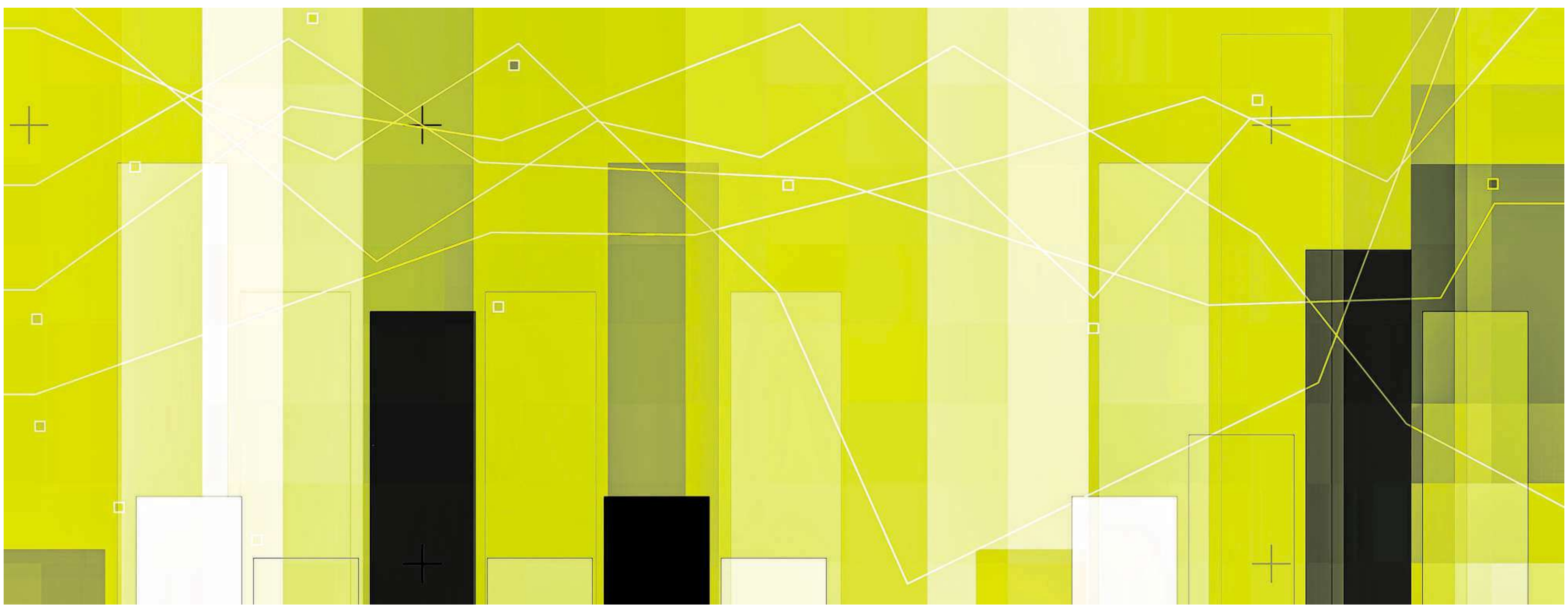
Answers:
(a) $\frac{13}{52} = \frac{1}{4}$ **(b)** $\frac{4}{52} = \frac{1}{13}$ **(c)** $\frac{20}{52} = \frac{5}{13}$

8. Two Events: Using a sample space

Q. A fair die is thrown and a coin is tossed.
(i) List the set of all possible outcomes i.e. the sample space.
(ii) Find the probability of getting a 3 and a Head.
Answer:
(i) {1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}
(ii) P(3H) = $\frac{1}{12}$

HIGHER
9. Relative Frequency also known as **experimental probability** is an estimate of the probability of an event happening.
$$\text{Relative Frequency} = \frac{\text{Number of successful trials}}{\text{Total number of trials}}$$

Q. A coin is flipped 1000 times. It lands on heads 450 times. Find the relative frequency of getting a head.
Answer: $\frac{450}{1000} = \frac{9}{20}$ or 0.45 or 45%



HIGHER

10. Expected Frequency = P(E) × number of trials.

Q. The probability that a biased die will land on each of the numbers 1 to 6 is given in the following table

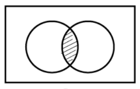
Number	1	2	3	4	5	6
Probability	0.2	x	0.3	0.1	0.2	0.1

(i) Write down the values of x.
(ii) If the die is thrown 500 times, how many fives would you expect?
Answers (i) $x = 0.1$ **(ii)** 100

11. HIGHER: Set Theory

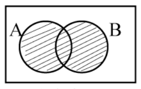
Learn: Shaded area

U



A ∩ B
"both A and B"

U



A ∪ B
"A or B"

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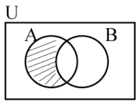
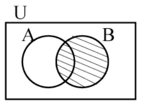


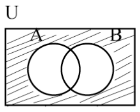
A \ B
"A only"

U



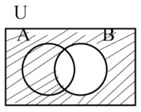
B

U



$(A \cup B)^{\prime}$

U



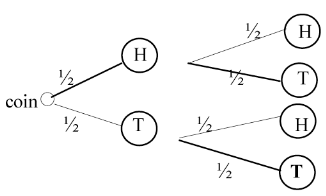
$B^{\prime}$

Remember: AND = ∩ ... intersection
OR = ∪ ... union

12. HIGHER: Tree Diagram

The possible outcomes of two or more events can be shown in a particular type of diagram called a **tree diagram.** This is sometimes referred to as a **probability tree**. The tree diagram below shows the outcomes and probabilities when a coin is tossed twice.

1st Toss 2nd Toss



Outcome	Probability
(H, H)	½ × ½ = ¼
(H, T)	½ × ½ = ¼
(T, H)	½ × ½ = ¼
(T, T)	½ × ½ = ¼
Total	1

We can see from the example above that the probability of getting a head on the first toss and a head on the second toss is
P(H, H) = P(Head) and P(Head)

= ½ × ½ = ¼

And means multiply

Notice that the sum of the probabilities at the end of the four branches adds up to 1.

Strand 2: Synthetic Geometry

Theorems and Constructions

A. Constructions:

Complete the constructions specified

1. Bisector of a given angle, using only compass and straight edge.
2. Perpendicular bisector of a line segment, using only compass and straight edge.
3. (H) Line perpendicular to a given line *l*, passing through a given point not on *l*.
4. Line perpendicular to a given line *l*, passing through a given point on *l*.
5. Line parallel to a given line, through a given point.
6. Division of a line segment into 2 or 3 equal segments, without measuring it.
7. (H) Division of a line segment into any number of equal segments, without measuring it.
8. Line segment of a given length on a given ray.
9. Angle of a given number of degrees with a given ray as one arm.
10. Triangle, given lengths of three sides.
11. Triangle, given SAS data.
12. Triangle, given ASA data.
13. Right-angled triangle, given the length of the hypotenuse and one other side.
14. Right-angled triangle, given one side and one of the acute angles
15. Rectangle, given side lengths.

B. Theorems:
Apply the results of all theorem, converses and corollaries to solve problems.

1. Vertically opposite angles are equal in measure.
2. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.
3. If a transversal makes equal alternate angles on two lines then the lines are parallel (and converse)
4. The angles in any triangle add to 180°
5. Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
6. Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
7. In a parallelogram, opposite sides are equal and opposite angles are equal (and converses).
8. The diagonals of a parallelogram bisect each other.
9. (H) If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.
10. (H) Let ABC be a triangle. If a line *l* is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio (and converse).
11. (H) If two triangles are similar, then their sides are proportional, in order (and converse) [statements only at OL].
12. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.
13. If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.
14. (H) The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Note: Formal proofs of theorem 4, 6, 9, 14 and 19 are examinable at higher level. Formal proofs are not examinable at ordinary level.

C. Corollaries:

1. (H) A diagonal divides a parallelogram into 2 congruent triangles.
2. (H) All angles at points of a circle, standing on the same arc, are equal, (and converse).
3. Each angle in a semi-circle is a right angle.
4. If the angle standing on a chord [BC] at some point of the circle is a right angle, then [BC] is a diameter.
5. (H) If ABCD is a cyclic quadrilateral, then opposite angles sum to 180°, (and converse).

E. Know the following terms

1. An Axiom is a statement applied without proof.
2. A theorem is a statement that can be shown to be true through the use of axioms and logical argument.
3. A corollary is a statement attached to a theorem which has been proven and follows obviously from it.
4, The converse of a theorem is the opposite or reverse of the theorem.
5. Implies means that when one result is established another result follows logically from it.

D. Transformation Geometry
·Transactions ·Central Symmetry ·Axial Symmetry
Learn how to
·Locate axes of symmetry in simple shapes
·Recognise images of points and objects under T, CS and AS.