Trigonometry Revision Sheet – Q5 of Paper 2

The Basics -

The Trigonometry section is all about triangles.

We will normally be given some of the sides or angles of a triangle and we use formulae and rules to find the others.
Some rules, such as the Tan, Cos and Sin ratios have to be learned while others such as the Sine Rule and Cosine Rule can be found in the tables.

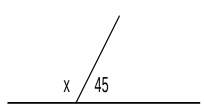
Page 9 of the tables contains many of the formulae you need, so use this page when practicing questions.

It is very important to practice questions to become familiar with when to use each rule.
There are basically 4 types of triangle we can be asked to solve.

1. Right angled triangles using the Tan, Cos, Sin ratios
2. Non-right angled triangles using Sine rule.
3. Double triangles using a combination of the above.
4. Areas of triangles and sectors using formula in the tables.

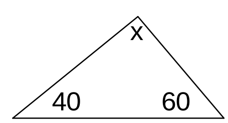
Things to remember:

Angles of a straight-line sum to 180



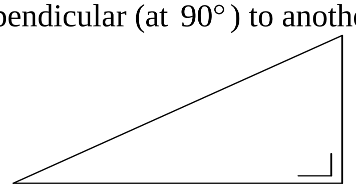
so in this example x = 180 – 45 = **135**

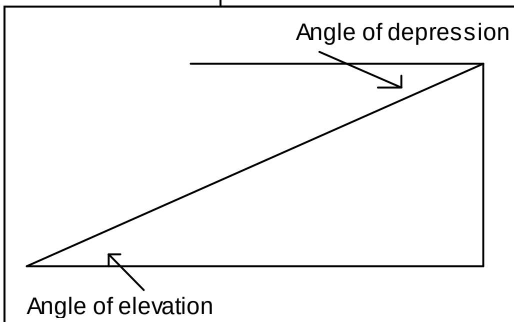
Angles of a triangle sum to 180

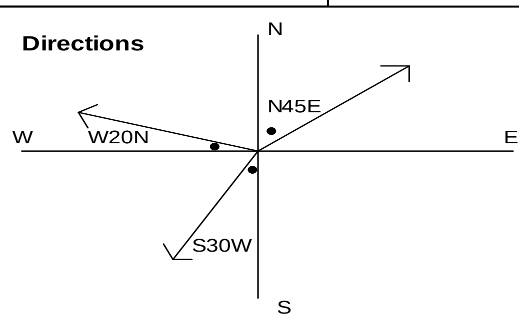


so in this example x = 180 – (60 + 40) = **80**

A right angle triangle is a triangle where one side is perpendicular (at $90°$) to another.





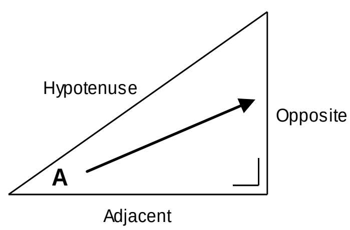


In all of the trigonometry questions the first step is always to draw a rough sketch of the triangle and put in an X on the side or angle you are looking to find.

Pythagoras -

In a right angle triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Basically the long side squared is equal to the squares of the other two sides added together.



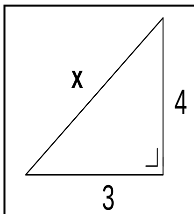
The hypotenuse is always the longest side of a right-angled triangle.
The opposite side is found by drawing an arrow through the angle.
The adjacent is the remaining side.

If we are given two sides of a right-angled triangle we can use Pythagoras to find the remaining side.

When answering any trigonometry question always your rough sketch and mark in the hypotenuse, the opposite and adjacent sides.

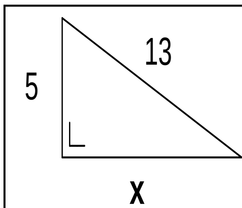
Find the value of x in the below examples.

Example 1



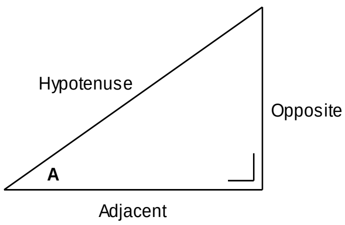
$$x^2 = 4^2 + 3^2$$
$$x^2 = 16 + 9$$
$$x^2 = 25$$
$$x = 5$$

Example 2



$$13^2 = 5^2 + x^2$$
$$169 = 25 + x^2$$
$$169 - 25 = x^2$$
$$144 = x^2$$
$$x = 12$$

The Tan, Sin and Cos Ratios –

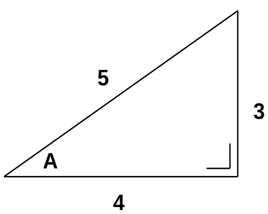


$TanA = \dfrac{Opposite}{Adjacent}$	$Toms = \dfrac{Old}{Aunt}$
$SinA = \dfrac{Opposite}{Hypotenuse}$	$Sat = \dfrac{On}{Her}$
$CosA = \dfrac{Adjacent}{Hypotenuse}$	$Coat = \dfrac{And}{Hat}$

These ratios are not in the tables and must be learned off by heart.

If we have one ratio (as a fraction) we can use it to find the other two. Always draw a rough sketch to help you work out what ratio to use.

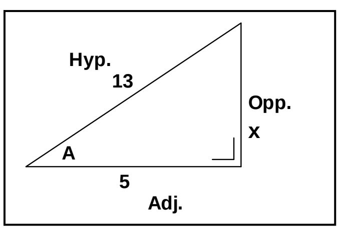
Example – Find Tan A, Cos A and Sin A of the following triangle.



$TanA = \dfrac{Opposite}{Adjacent}$	$TanA = \dfrac{3}{4}$	*1. Label the sides opposite, adjacent and hypotenuse*
$SinA = \dfrac{Opposite}{Hypotenuse}$	$SinA = \dfrac{3}{5}$	*2. Write down the ratios.*
$CosA = \dfrac{Adjacent}{Hypotenuse}$	$CosA = \dfrac{4}{5}$	*3. Insert the relevant numbers.*

Sometimes all the sides will not be given to you and you must work them out using Pythagoras.

Example – $CosA = \dfrac{5}{13}$ Find $TanA$ and $SinA$



$$CosA = \frac{5}{13} \qquad CosA = \frac{Adj.}{Hyp.}$$

Adjacent = 5
Hypotenuse = 13

so we draw a rough sketch using 5 as the adjacent side and 13 as the hypotenuse.

We now use Pythagoras to find the missing side, X.

$$13^2 = 5^2 + x^2$$
$$169 = 25 + x^2$$
$$169 - 25 = x^2$$
$$144 = x^2$$
$$x = 12$$

Opposite = 12

$$TanA = \frac{Opposite}{Adjacent} \qquad \boxed{TanA = \frac{12}{5}}$$

$$SinA = \frac{Opposite}{Hypotenuse} \qquad \boxed{SinA = \frac{12}{13}}$$

Calculator Work –

You need to be very comfortable using your calculator way before the exam as they differ greatly from model to model.

Always make sure your calculator is in DEG mode.

There are two main things we need to be able to do:

Find **Tan, Sin or Cos** of a given angle.

Example – Find Cos 53°

1. Press Cos
2. then type 53
3. then '='

= **0.6018** (round off to 4 decimals)

Example – Find Sin 33°16'

1. Press Sin
2. then type 33
3. then the DMS button
4. then 16
5. then '='

= 0.5486 (round off to 4 decimals)

Find the **angle** given the Tan, Sin or Cos.

Example – Cos A = 0.7071 Find A

1. Press 2nd Function
2. Press Cos
3. then type 0.7071
4. then '='

A = 45°

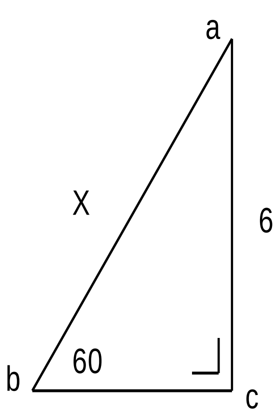
If you get a decimal answer you can change it into degrees and minutes by again pressing the DMS button OR the 2nd Function then the DMS button.

Solving right-angled triangles -

We can use our ratios Tan, Sin and Cos to find missing sides or angles of right-angled triangles.

1. To do this we first draw a rough sketch of the triangle filing in any information we know.
2. We put an X next to the angle or side we are looking for.
3. We decide what ratio Tan, Sin or Cos is relevant.
4. We solve for x.

Example – abc is a right-angled triangle with |ac|= 6cm and $\angle abc = 60°$. Find |ab|.



We first draw a rough sketch labelling the sides and placing an X on the side we are looking for.
*The **o**pposite (6) and **h**ypotenuse (X) are the sides involved so we use the Sin ratio.*

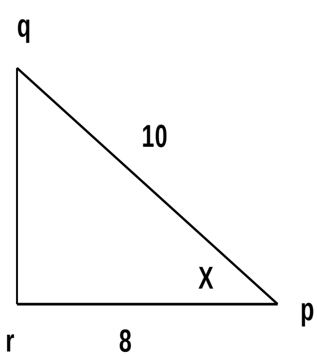
$SinA = \frac{Opposite}{Hypotenuse}$ — *Write down the formula*

$Sin60 = \frac{6}{x}$ — *Insert the known values*

$x = \frac{6}{Sin60}$ — *Cross multiply*

$x = \frac{6}{0.8660} = 6.93cm$ — ***The value of |ab|***

Example – pqr is a right-angled triangle with |pr| = 8 and |pq| = 10. Calculate the value of $\angle pqr$.



We first draw a rough sketch labelling the sides and this time placing an X in the andle we are looking for.
*The **a**djacent (8) and **h**ypotenuse (10) are the sides involved so we use the Cos ratio.*

$CosA = \frac{Adjacent}{Hypotenuse}$ — *Write down the formula*

$CosX = \frac{8}{10}$ — *Insert the known values*

$CosX = 0.8000$ — *Change fraction into decimal*

$X = \boxed{2^{nd}\ Function}\ \boxed{Cos}\ \boxed{0.8000}$ — *Use calculator to find Angle*

$X = 36°52'$ — ***The angle*** $\angle pqr$

Angles of 30, 45, 60 and 90.

Some questions require answers to be left in fraction form **without** the use of a calculator.

Any time you see the Sin, Cos or Tan of the angles of 30, 45, 60 or 90 you can look up their fraction/ surd values in the tables and insert them into the question.

In the tables on page 9, π stands for $180°$.

A	0	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
Cos A	1	-1	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Sin A	0	0	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tan A	0	0		$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

Example – Find in fraction form

1. $\sin 45 \cos 45 + \cos^2 30$
2. $\tan^2 30 + \sin^2 60$

1. $\sin 45 \cos 45 + \cos^2 30$

$$\frac{1}{\sqrt{2}} . \frac{1}{\sqrt{2}} + (\frac{\sqrt{3}}{2})^2$$

$$\frac{1}{2} + \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2}$$

$$\frac{1}{2} + \frac{3}{4}$$

$$\frac{1(2) + 3(1)}{4}$$

$$\frac{5}{4}$$

2. $\tan^2 30 + \sin^2 60$

$$(\frac{1}{\sqrt{3}})^2 \times (\frac{\sqrt{3}}{2})^2$$

$$\frac{1}{\sqrt{3}} . \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2}$$

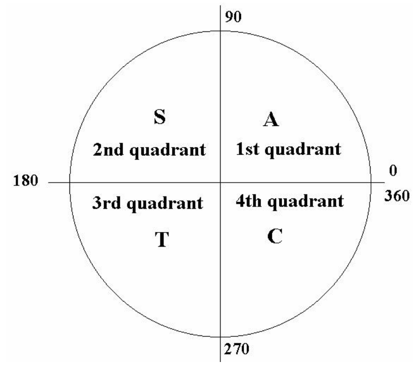
$$\frac{1}{3} \times \frac{3}{4}$$

$$\frac{3}{12}$$

$$\frac{1}{4}$$

Angles greater than 90 - The Unit Circle

For questions involving angles greater than $90°$ we use the unit circle.
The letter in each quadrant determines whether Sin, Cos or Tan is negative or positive in that quadrant.
Between 0 and 90 – **A**ll are positive
Between 90 and 180 – **S**in is positive, Tan, Cos negative
Between 180 and 270 – **T**an is positive, Sin, Cos negative
Between 270 and 360 – **C**os is positive, Sin, Tan negative

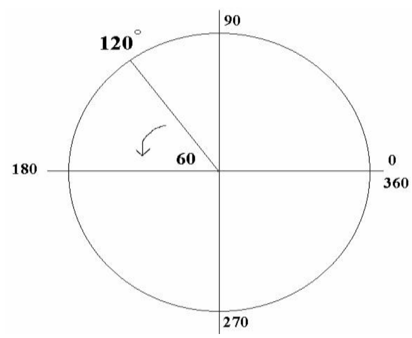


To find the ratio of an angle greater than 90 we use the following steps.

1. Make a rough sketch of the angle in the unit circle.
2. Use 

to find if the ratio is positive or negative.
3. Find the reference angle (the angle between your sketched line and the x axis).
4. Use a calculator to find required ratio of this reference angle and use the sign from step 2.

Example – Find the value of $Sin 30°$.



Draw a rough sketch of $120°$.
In the 2nd quadrant Sin is ***positive.***
The reference angle is $(180° - 120°) = 60°$

Use your tables to find the $Sin 60 = \frac{\sqrt{3}}{2}$

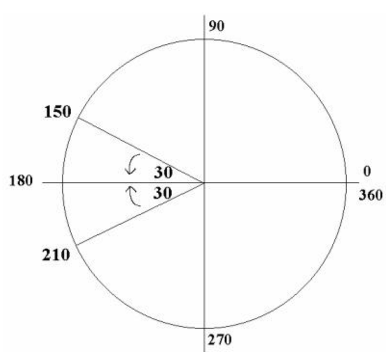
Use the ***positive*** sign from above $= \frac{\sqrt{3}}{2}$

To find the angles (there will be two) given the ratio we use the following steps.

1. Ignore the sign and use your calculator to find the reference angle.
2. Look at the sign and use 

to see what quadrants your two angles lie in.
3. Draw the two angles – they will be the reference angle distance from the x axis.

Example – Find the two values of A, given that Cos A = -0.8660 and $0° \le A \le 360°$

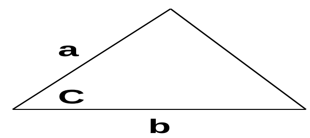


Ignoring the sign use your calculator to find the reference angle using 2nd $\boxed{\text{Function}}$ $\boxed{\text{Cos}}$ $\boxed{0.8660}$ = $30°$
-0.8660 is **negative** and Cos is negative in the 2nd and 3rd quadrants.
Draw the two angles:
In the 2nd quadrant $180° - 30° = 150°$
In the 2nd quadrant $180° + 30° = 210°$

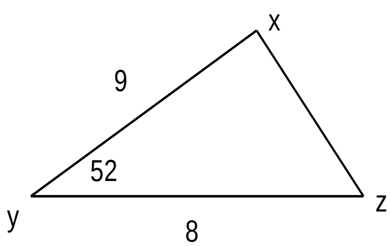
Area of a triangle – The area of a triangle can be found using the following formula (Page 9 of the tables)

$\boxed{\frac{1}{2}abSinC}$

where **a** and **b** are the sides of a triangle and **C** is the angle between them.



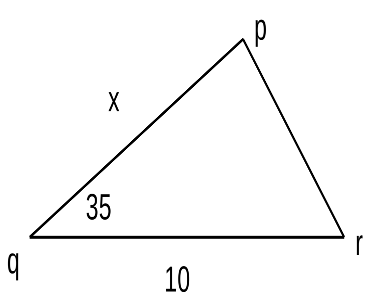
Example – Find the Area of triangle xyz



$\frac{1}{2}abSinC$	*Write down formula*
$=\frac{1}{2}(9)(8)Sin52°$	*Insert values*
$=\frac{1}{2}(9)(8)(.7880)$	*Work out $Sin52°$*
$=28.37cm^2$	*Area of the triangle*

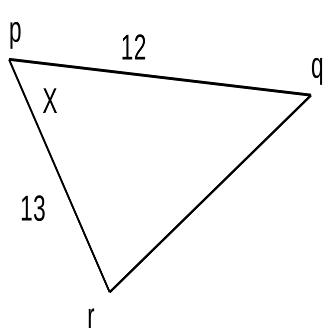
If we are given the area we can use the formula to find a side or angle.

Example – The Area of triangle pqr is $24cm^2$. Find the length of the side |pq|



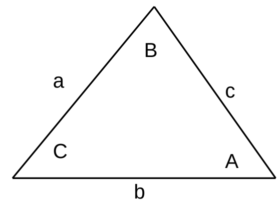
$\frac{1}{2}abSinC=24$	*Write down formula equal to 24*
$\frac{1}{2}(x)(10)Sin35°=24$	*Insert values including x*
$\frac{1}{2}(x)(10)(.5736)=24$	*Work out $Sin35°$*
$2.87x=24$	*Simplify*
$x=\frac{24}{2.87}=8.37cm^2$	*Divide across by x to find the side*

Example – The Area of triangle pqr is $50cm^2$. Find the angle of $\angle$ qpr



$\frac{1}{2}abSinC=50$	*Write down formula equal to 50*
$\frac{1}{2}(12)(13)SinX=50$	*Sub in known value*
$78SinX=50$	*Simplify*
$SinX=\frac{50}{78}$	*Divide across by 78*
$SinX=0.6410$	*50 divided by 78 = 0.6410*
$X=\boxed{2^{nd}\ Function}\ \boxed{Sin}\ \boxed{0.6410}=39°52'$	*The angle* $\angle$ qpr

Sine Rule – $\boxed{\dfrac{a}{SinA} = \dfrac{b}{SinB}}$



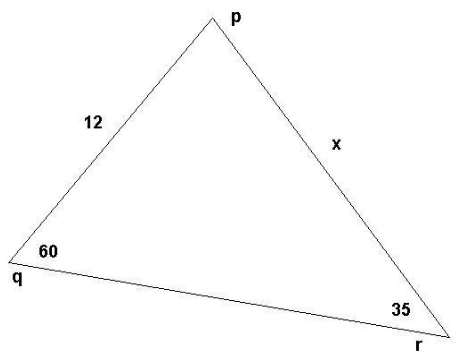
Tables Page 9

We use the Sine formula to find a missing side or angle when we have been given 2 sides and one of the opposite angles **OR** two angles and one of the opposite sides of a triangle.

The formula basically says that

$$\frac{\text{Any side}}{\text{The Sine of the opposite angle}} = \frac{\text{Any other side}}{\text{The Sine of its opposite angle}}$$

Example – In the triangle pqr, $|pq| = 12cm$, $\angle pqr = 60°$ *and* $\angle qrp = 35°$. *Find* $|pr|$.



$\dfrac{a}{SinA} = \dfrac{b}{SinB}$ *Write down formula*

$\dfrac{12}{Sin35} = \dfrac{x}{Sin60}$ *Insert known values*

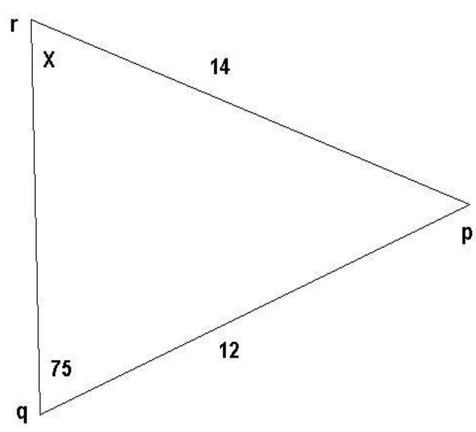
$x \sin 35 = 12 \sin 60$ *Cross Multiply*

$x(0.5736) = 12(0.8660)$ *Calculator for Sine's*

$x = \dfrac{12(0.8660)}{0.5736} = \dfrac{10.39}{0.5736}$ *Divide across by 0.5736*

x = 18.11 $\boxed{\mathbf{|pr| = 18.11cm}}$

Example – In the triangle pqr, $|pq| = 12cm$, $|pr| = 14cm$ *and* $\angle pqr = 75°$. *Find the* $\angle prq$



$\dfrac{a}{SinA} = \dfrac{b}{SinB}$ *Write down formula*

$\dfrac{14}{Sin75} = \dfrac{12}{SinX}$ *Insert known values*

$14 \sin X = 12 \sin 75$ *Cross Multiply*

$SinX = \dfrac{12 \sin 75}{14}$ *Divide across by 14*

$SinX = \dfrac{12(0.9659)}{14} = \dfrac{11.59}{14} = 0.8279$ *Simplify*

Sin X = 0.8279

X = $\boxed{2^{nd}\ Function}$ $\boxed{Sin}$ $\boxed{0.8279}$ *= 55.88* *Calculator*

$\boxed{X = 55°52'}$ ***Value of*** $\angle prq$

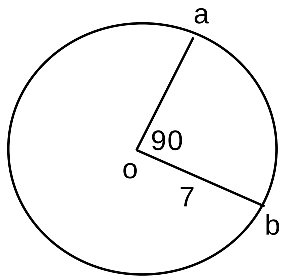
Area of a Sector -

A sector is a part or slice of a circle. To find the area of a sector we actually use information from the Area and Volume question.

Remember that the area of a circle is πr^2
To find the sector (which is only a fraction of the circle) we multiply πr^2 by the fraction.

Therefore $\boxed{\text{Area of a Sector} = \frac{\phi}{360} \times \pi r^2}$ where ϕ is the angle of the sector

Example – Find the area of the sector aob where o is the centre of a circle with radius 7.
Use π = 3.14



$\frac{\phi}{360} \times \pi r^2$ *Write down the formula*

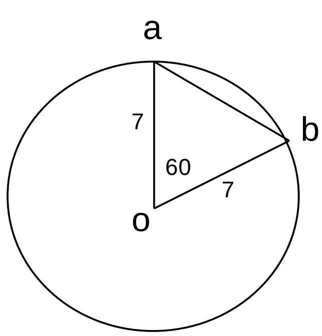
$\frac{90}{360} \times (3.14)(7)^2$ *Sub in known values*

$0.25 \times (3.14)(49)$ *Simplify*

$= 38.5cm^2$ *Area of the sector*

Some questions involve using the area of sector formula along with the area of a triangle formula.

Example – Find the shaded area in the circle with centre o and radius 7.
Use $\pi = \frac{22}{7}$



Area of shaded region = Area of sector – Area of triangle

Area of sector = $\frac{\phi}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7}(7)^2$

$= \frac{60}{360} \times \frac{22}{7}(7)^2$

$= 0.17 \times 154$

$= 26.18cm^2$

Area of triangle = $\frac{1}{2} abSinC$

$= \frac{1}{2}(7)(7)Sin60$

$= \frac{1}{2}(7)(7)(0.87)$

$=0.5(7)(7)(0.87)$

$= 21.32cm^2$

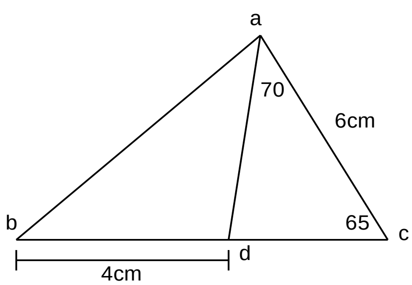
Area of shaded region = Area of sector – Area of triangle

$= 26.18cm^2$ - $21.32cm^2$ = **4.86** cm^2

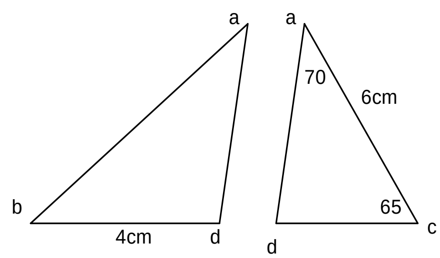
Double triangles –

The rules for solving double triangles are the same as those in the previous sections. Mostly they will be two-part questions, one part asking you to solve a small triangle within the large one using the cosine or sine rule and the other part asking you to find an angle or side on the larger triangle.

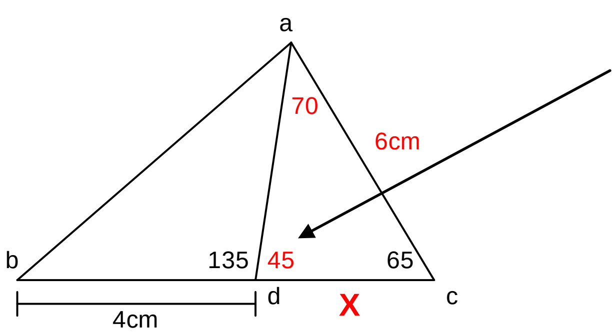
Example – In the triangle abc, |bd| = 4, |ac| = 6, $\angle dac = 70°$ and $\angle dca = 65°$
(i) Find |dc| **(ii)** Find |ab|



Notice that this is what we mean by a double triangle. Two smaller triangles adc and abd make up the larger triangle abc.



The first thing we do is fill in any missing information we can knowing that straight lines add up to $180°$ and the three angles in a triangle add to $180°$.



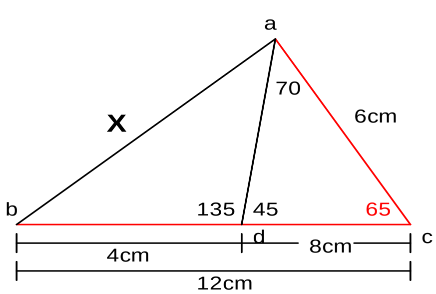
180 – (70 + 65) = 45 *(3 angles add to 180)*
Fill in X for the missing value |dc|
*Now we can use **Sine Rule** with the triangle adc*

$$\frac{a}{SinA} = \frac{b}{SinB} \longrightarrow \frac{6}{Sin45} = \frac{x}{Sin70}$$

x(Sin45) = 6(Sin70)
x(0.7071) = 6(.9397)

$$x = \frac{6(.9397)}{0.7071} = \mathbf{8cm}$$

$\boxed{|dc| = 8cm}$ ***(i)***



If we put 8cm in for |dc| we can see that the total length of |bc| is 12cm.
*Now we can use the **Cosine rule** with triangle abc*

$$a^2 = b^2 + c^2 - 2bcCosA$$
$$x^2 = 6^2 + 12^2 - 2(6)(12)Cos60$$
$$x^2 = 36 + 144 - 2(6)(12)(0.5)$$
$$x^2 = 36 + 144 - 72$$
$$x^2 = 108$$

x = 10.4cm $\boxed{|ab| = 10.4cm}$ **(ii)**

If were asked we could also find the area of the triangles abc or adc using the formula.