The Line Revision Sheet – Questions 2 Paper 2

We deal with these topics together as many of the concepts are we learn for the line such as distances, slopes, midpoints, equations of lines are needed for the circle question

THE FORMULAE

Given two points $(x_{1,}y_1)$ and $(x_2 y_2)$

Distance $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Slope $\frac{y_2 - y_1}{x_2 - x_1}$

If we have a line $ax + by + c = 0$ then

$m = -\frac{a}{b}$

Equation of a line (or tangent) with a slope **m** and point $(x_{1,}y_1)$

$y - y_1 = m(x - x_1)$

The area of a triangle with vertices (0, 0) $(x_{1,}y_1)$ and $(x_2 y_2)$

$\boxed{\frac{1}{2} | x_1 y_2 - x_2 y_1 |}$

You must learn off by heart all of the above formulae before you go into the exam, as they do **NOT** appear in the Tables.

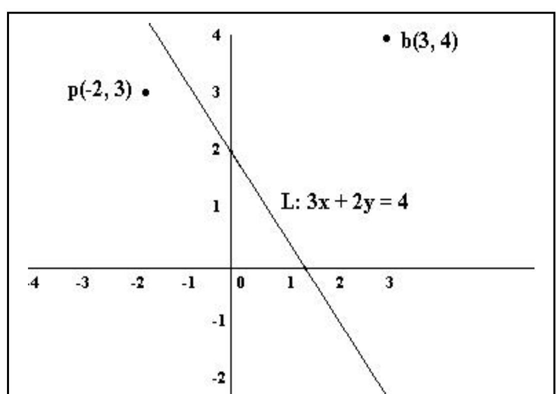
Notation –

Every point has two values, an x value and a y value, and can be placed on a graph using an x-axis and a y-axis. We use small letters to denote a point.

b(3, 4) denotes the point b which we find by going to the right 3 units on the x axis and up 4 units on the y axis.

p(-2, 3) denotes the point p which we find by going to the left 2 units and up 3 units.

We use capital letters to denote a line. L: 3x + 2y = 4
In this case L is the line 3x + 2y = 4



To find the Distance between two points

Step 1 – Label your points (x_1, y_1) and (x_2, y_2)

Step 2 – Write down the distance formula - $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Step 3 – Fill points into formula and work out.

Example – a(3, 4) and b(6, -1) are two points, find the distance between them |ab|

(3, 4)	(6, -1)	*Write down the points*
(x_1, y_1)	(x_2, y_2)	*Label them*

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	*Write down the formula*
$\sqrt{(6-3)^2 + (-1-4)^2}$	*Put in the values for x and y*
$\sqrt{(3)^2 + (-5)^2}$	*Simplify*
$\sqrt{9+25}$	*Simplify*
$\sqrt{34}$	*Simplify*

To find the Midpoint –

Step 1 – Label your points (x_1, y_1) and (x_2, y_2)

Step 2 – Write down the distance formula - $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Step 3 – Fill points into formula and work out.

Midpoint - $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Example – a(3, 4) and b(6, -1) are two points, find the midpoint of ab

(3, 4)	(6, -1)	*Write down the points*
(x_1, y_1)	(x_2, y_2)	*Label them*

$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$	*Write down the formula*
$(\frac{3+6}{2}, \frac{4+(-1)}{2})$	*Put in the values for x and y*
$(\frac{9}{2}, \frac{3}{2})$	*Simplify*
$(4.5, 1.5)$	*Simplify*

To find the slope –

We can find the slope in two ways.

1. If we have two points we find the slope between them by using the slope formula.

2. If we have a line $ax + by + c = 0$ then $m = -\frac{a}{b}$

Slope $\frac{y_2 - y_1}{x_2 - x_1}$

Example – a(3, 4) and b(6, -1) are two points, find the slope of ab

(3, 4)	(6, -1)	*Write down the points*
(x_1, y_1)	(x_2, y_2)	*Label them*
$\frac{y_2 - y_1}{x_2 - x_1}$		*Write down the formula*
$\frac{-1-4}{6-3}$		*Put in the values for x and y*
$\frac{-5}{3}$		*Simplify*

Important things to remember.

- If two lines are parallel their slopes are equal.
- If two lines are perpendicular then their slopes multiply to give –1. If you are given one slope turn it upside down and change the sign to find the perpendicular slope.

Example – What is the slope of a line perpendicular to 3x + 5y +7 =0

3x + 5y +7 =0	*Write down the line*
$ax + by + c = 0$ then $m = -\frac{a}{b}$	*Use* $m = -\frac{a}{b}$ *formula to find slope*
$m = -\frac{3}{5}$	
$m = -\frac{3}{5}$ then perpendicular to $m = +\frac{5}{3}$	*Turn upside down and change sign to find perpendicular slope*

If two lines are perpendicular they will equal –1 when you multiply them.

From above $\frac{5}{3} \times -\frac{3}{5} = -\frac{15}{15} = -1$

To find the equation of a line –

We use the formula below to write the equation of a line.

To use the below equation we must have:

1. A point on the line
2. The slope of the line

If we are given two points on the line we can use the slope formula to find its slope and then use either point in the formula.

Equation of a line (or tangent) with a slope **m** and point $(x_{1,} y_1)$

$y - y_1 = m(x - x_1)$

Example – Find the equation of a line passing through the point (3, 4) with a slope –3

(3, 4)	-3	*Write down the point and the slope*
(x_1, y_1)	m	*Label the point and the slope*

$y - y_1 = m(x - x_1)$	*Write down the formula*
$y - 4 = -3(x - 3)$	*Put in the values for x, y and m*
$y - 4 = -3x + 9$	*Simplify*
$3x + y - 4 - 9 = 0$	*Bring everything to one side*
$3x + y - 13 = 0$	*Simplify*

Example – Find the equation of the line passing through the points (4, 2) and (-1, 3)

(4, 2)	(-1, 3)
(x_1, y_1)	(x_2, y_2)

$\dfrac{y_2 - y_1}{x_2 - x_1}$	*We must first find the slope*
$\dfrac{3 - 2}{-1 - 4}$	*Put in the values for x and y*
$\dfrac{1}{-5} = -\dfrac{1}{5}$	*Simplify to get slope m* $= -\dfrac{1}{5}$

$y - y_1 = m(x - x_1)$	*Write down the formula*
$y - 2 = -\dfrac{1}{5}(x - 4)$	*Put in the values for x, y and m*
$5(y - 2) = -1(x - 4)$	*We multiply across by 5*
$5y - 10 = -x + 4$	*Simplify*
$x + 5y - 10 - 4 = 0$	*Bring everything to one side*
$x + 5y - 14 = 0$	*Simplify*

Example – K is the line 3x - 2y +7 = 0. Find the equation of the line, M that is perpendicular to K and passes through the point (4, -2)

3x - 2y +7 = 0	*Write down the line K*
$ax+by+c=0$ then $m=-\frac{a}{b}$	*Use* $m=-\frac{a}{b}$ *formula to find slope*
$m=-\frac{3}{-2}=\frac{3}{2}$	*minus by a minus is a plus*
$m=\frac{3}{2}$ then perpendicular to $m=-\frac{2}{3}$	*Turn upside down and change sign to find perpendicular slope*
(4, -2) $\quad -\frac{2}{3}$	*Write down the point and the slope*
(x_1, y_1) $\quad$ m	*Label the point and the slope*
$y-y_1=m(x-x_1)$	*Write down the formula for line*
$y-(-2)=-\frac{2}{3}(x-4)$	*Put in the values for x, y and m*
$3(y+2)=-2(x-4)$	*We multiply across by 3*
$3y+6=-2x+8$	*Simplify*
$2x+3y+6-8=0$	*Bring everything to one side*
$2x+3y-2=0$	*Simplify*

To find the Area of a triangle –

To find the area of a triangle we need to know the points of the three vertices of the triangle. One of these vertices MUST be (0, 0) and then we label the others (x_1, y_1) and (x_2, y_2) to put into the formula below. If one of the vertices is not (0, 0) then we must move one of the points so that it is (0, 0). We then move the other two points the same distance.

Area of a triangle – $\boxed{\frac{1}{2}|x_1y_2 - x_2y_1|}$

Example – (0, 0) (3, 4) and b(6, -1) are the vertices of a triangle. Find the area of the triangle.

(0, 0)	(3, 4)	(6, -1)	*Write down the points*
	(x_1, y_1)	(x_2, y_2)	*Label them*

$\frac{1}{2}|x_1y_2 - x_2y_1|$ — *Write down the formula*

$\frac{1}{2}|(3)(-1) - (6)(4)|$ — *Put in the values for x and y*

$\frac{1}{2}|-3 - 24|$ — *Simplify*

$\frac{1}{2}|-27|$ — *Simplify*

13.5 — *Simplify (always positive answer)*

Example – (2, 4) (3, -3) and b(-3, 1) are the vertices of a triangle. Find the area of the triangle.

(2, -4) $\longrightarrow$ (0, 0)
(3, -3) $\longrightarrow$ (1, 1) (x_1, y_1)
(-2, 1) $\longrightarrow$ (-4, 5) (x_2, y_2)

> None of the points were (0, 0) so we must move one of them to (0, 0) and then the rest the same distance.
> (2, -4) $\longrightarrow$ (0, 0) means we move the x value down 2 from 2 to 0 and the y value up 4 from –4 to 0
> So for each of the points we move the x value down 2 and the y value up 4.

$\frac{1}{2}|x_1y_2 - x_2y_1|$ — *Write down the formula*

$\frac{1}{2}|(1)(5) - (1)(-4)|$ — *Put in the values for x and y*

$\frac{1}{2}|5 + 4|$ — *Simplify*

$\frac{1}{2}|9|$ — *Simplify*

4.5 — *Simplify (always positive answer)*

To show a point is on a line –

Put the x and y co-ordinates of the point into the line for x and y and if it returns a true statement then that point IS on the line.

Example – Investigate if the point (3, 4) is on the line $3x + y - 13 = 0$

$3x + y - 13 = 0$	*Write down the line*
$3(3) + (4) - 13 = 0$	*Put in the values for x and y*
$9 + 4 - 13 = 0$	*Simplify*
$13 - 13 = 0$	*Simplify*

TRUE *Statement is true therefore (3, 4) is on the line* $3x + y - 13 = 0$

Example – Investigate if the point (-2, 4) is on the line $3x + y - 13 = 0$

$3x + y - 13 = 0$	*Write down the line*
$3(-2) + (4) - 13 = 0$	*Put in the values for x and y*
$-6 + 4 - 13 = 0$	*Simplify*

FALSE *Statement is false therefore (-2, 4) is NOT on the line* $3x + y - 13 = 0$

To move a point under a given translation –

To translate a point means to move it from one place to another.
There are a number of ways they can ask you to move points.

1. Given a rule
2. Through Axial symmetry
3. Through Central Symmetry

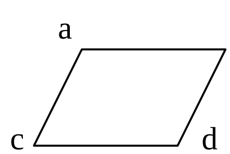
Given a rule – This is very similar to moving the points of a triangle as on page 6.

Example – Find the image of the point (3, 1) through the translation (2, -1) $\longrightarrow$ (4, 1)

(2, -1) $\longrightarrow$ (4, 1) *Write down the translation*
x value moved up 2, y value moved up 2

(3, 1) $\longrightarrow$ *(5, 3)* *move x value 3 up 2, move y value 1 up 2*

Example – a(-3, -1) b(-1, 0) c(0, -3) and d (x, y) are four points of the parallologram abcd. Find the values of x and y.



Draw a rough sketch

We can see from our sketch that going from c to d would be the same direction and distance as going from a to b.

a(-3, -1) $\longrightarrow$ b(-1, 0) *Write down the translation a to b*

c(0, -3) $\longrightarrow$ d(2, -2) *Move c the same distance*
(x value up 2 and y value up 1)

(x, y) = (2, -2)

Axial symmetry – This means moving a point through a line(axis) at a right angle and the same distance out the other side. There are three types.

S_x **Axial Symmetry in the x axis just change the sign of y co-ordinate**

S_y **Axial Symmetry in the y axis just change the sign of x co-ordinate**

S_o **Central Symmetry through the origin (0, 0) you are going through both axis just change sign of x and y co-ordinates.**

Example – Find the image of (2, -3) under (i) S_x (ii) S_y and (iii) S_o

S_x **(2, -3)** $\longrightarrow$ (2, 3) Change y sign

S_y **(2, -3)** $\longrightarrow$ (-2, -3) Change x sign

S_o **(2, -3)** $\longrightarrow$ (-2, 3) Change both signs

Central Symmetry – This means moving a point through a point and out the same distance and direction the other side.

Example – Find the image of (3, 1) through the point (1, 2)

(3, 1) $\longrightarrow$ (1, 2) *Write down the move from (3, 1) to (1, 2)*
x value went down 2, y value went up 1

(3, 1) $\longrightarrow$ (1, 2) $\longrightarrow$ (-1, 3) *Move the point the same distance past (1, 2)*
x value goes down 2 more, y value goes up 1 more

(-1, 3) is the image of (3, 1) through (1, 2)

To move a line through a given translation –

Step 1 – Find the slope of the line
Step 2 – Find a point on the line (do this by letting x = 0)

Step 3 – Find the image of this point
Step 4 – Use the equation of line formula with this new point and the slope found above

Example – Find the line K which is the image of the line L: 3x – 4y –8= 0 under the translation $(3, 1) \longrightarrow (1, 2)$

3x – 4y –8= 0	*Write down the line L*
$ax+by+c=0$ then $m=-\frac{a}{b}$	*Use* $m=-\frac{a}{b}$ *formula to find slope*
$m=-\frac{3}{-4}=\frac{3}{4}$	*The slope of the line L*

3x – 4y –8= 0	*Write down the line L*
3(0) –4y – 8= 0	*To find a point on L let x = 0*
-4y = 8	*y's to one side, numbers to the other*
4y = -8	*change signs*
$y=\frac{-8}{4}=-2$	*y co-ordinate*
(0, -2)	**A point on the line L**

We must move this point under the translation $(3, 1) \longrightarrow (1, 2)$

$(3, 1) \longrightarrow (1, 2)$	*Write down the translation*
$(0, -2) \longrightarrow (-2, -1)$	*Move x value down 2, y value up 1*

(-2, -1) is a point on the Line K

(-2, -1)	$\frac{3}{4}$	*Write down the point and the slope*
(x_1, y_1)	m	*Label the point and the slope*

$y-y_1=m(x-x_1)$	*Write down the equation of a line*
$y-(-1)=\frac{3}{4}(x-(-2))$	*Put in the values for x and y*
$y+1=\frac{3}{4}(x+2)$	*Simplify*
$4(y+1)=3(x+2)$	*Multiply across by 4*
$4y+4=3x+6$	*Simplify*
$-3x+4y-2=0$	*Equation of a line*

To draw a sketch of a line/ identify where a line intersects the x and y axis –

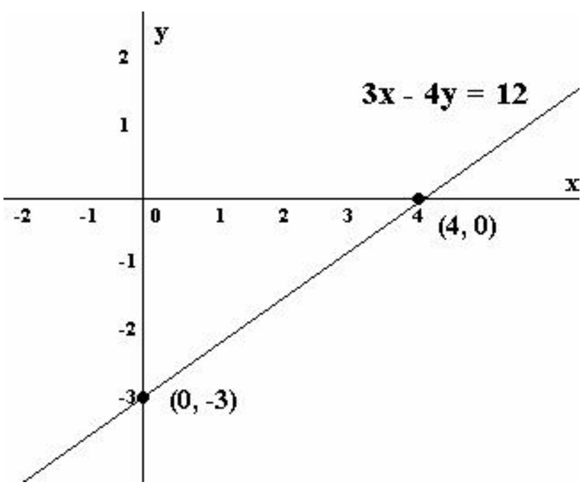
To draw a line we need to find at least two points on the line. The easiest points of a line to find are those where the line crosses the x-axis and the y-axis.

A line crosses the x axis at y = 0
A line crosses the y axis at x = 0

Learn this

Example – Draw a sketch of the line 3x – 4y = 12

	$3x - 4y = 12$	*Write down the line*
let y = 0	$3x - 4(0) = 12$	*Sub in y = 0*
	$3x = 12$	*Simplify*
	$x = \frac{12}{3}$	*Divide across by 3*
	$x = 4$	*x value when y = 0*
(4, 0)		*The point where the line cuts the x-axis*
let x = 0	$3(0) - 4y = 12$	*Write down the line*
	$-4y = 12$	*Sub in y = 0*
	$4y = -12$	*Change signs*
	$y = \frac{-12}{4}$	*Divide across by 4*
	$y = -3$	*y value when x = 0*
(0, -3)		The point where the line cuts the y-axis



Make sure to label the points, the line and the axis x and y.

To find where to lines intersect each other –

The find the point of intersection of two lines we do a simultaneous equation.

Example – Identify the point where the lines $5x+2y=7$ and $2x-y=10$

$5x+2y=7$ **Equation 1**
$2x-y=10$ **Equation 2**

We write one equation directly above the other.

$5x+2y=7$ **Equation 1**
$4x-2y=20$ **Equation 2**

We multiply the bottom equation by 5 to get the y values the same.

$5x+\cancel{2y}=7$ **Equation 1**
$4x\cancel{-2y}=20$ **Equation 2**
$9x \quad = \quad 27$

To cancel the y's their signs must be which they are so they can be cancelled

We then add or subtract the x's *5x + 4x = 9x*
We add or subtract the numbers *7 + 20 = 27*

$x=\frac{27}{9}=3$ *Divide across by 9 to get our* ***x value***

To get our **y value** we sub in our x value into either Equation 1 or Equation 2

$5x+2y=7$	**Equation 1**
$5(3)+2y=7$	*Sub in x value*
$15+2y=7$	*Simplify*
$2y = 7 - 15$	*y's to one side, numbers to the other*
$2y = -8$	*Simplify*
$y=-\frac{8}{2}=-4$	*the* ***y value***

x = 3 AND y = -4

The lines intersect at (3, -4)