

# Arithmetic Revision Sheet - Questions 1 and 2 of Paper 1

## Basics -

**Factors/ Divisors** – Numbers that divide evenly into a number.

Factors of 12 – 1, 2, 3, 4, 6, 12

Factors of 18 – 1, 2, 3, 6, 9, 18

**Highest Common Factor** of 12 and 18 is 6

**Multiples** – A multiple of a natural number is a number into which it divides evenly.

Multiples of 3 – 3, 6, 9, 12.....

Multiples of 4 – 4, 8, 12, 16.....

**Lowest Common Multiple** of 3 and 4 is 12

**Prime Numbers** – A numbers whose factors are only one and itself.

2, 3, 5, 7, 11, 13, 17, 19, 23.....

**Reciprocal** – This is a fraction turned upside down. To calculate the value of a reciprocal divide the bottom number into the top number and round off to two decimal places.

Reciprocal of  $\frac{7}{2}$  is  $\frac{2}{7}$  or 0.29

## Scientific Notation -

We often write very large or very small numbers in the form  $a \times 10^n$  where a is between 1 and 10

**Examples -**  $230,000 = 2.3 \times 10^5$

$6.1 \times 10^5 = 610,000$

$47,000,000 = 4.7 \times 10^7$

$1.5 \times 10^2 = 150$

$0.0078 = 7.8 \times 10^{-3}$

$2.6 \times 10^{-3} = 0.0026$

**Example** – Calculate  $(5.3 \times 10^4) + (2.6 \times 10^3)$  and give your answer in scientific notation.

$(5.3 \times 10^4) + (2.6 \times 10^3)$

*write equation*

$= 53000 + 2600$

*put in normal notation*

$= 55600$

*add together*

$= 5.56 \times 10^4$

*put back in scientific notation*

## Measurement -

If asked a question where measurements are in different units it is important to change them all into the same unit before attempting to solve.

Length	Mass	<i>These are the most common but others can be found in your tables if asked</i>
10mm = 1cm	1000mg = 1 g	
100cm = 1m	1000g = 1kg	
1000m = 1km	1000kg = 1 tonne	

## Percentages –

Percentages can come up in a variety of questions (Profit and Loss, VAT, Currency etc).

**To get a % of a number multiply the number by the percentage over 100.**

**Example** – Calculate VAT of 21% on a Fridge costing €630

$$= 630 \times \frac{21}{100} = \frac{13,230}{100} = 132.30$$

*We want to calculate 21% of €630*

Total cost of Fridge = €630 + €132.30 = €762.30

**To express one number as a % of another put one over the other and multiply by 100**

**Example** – Express 35cm as a percentage of 2m *(convert 2m to 200cm first)*

$$= \frac{35}{200} \times 100 = \frac{3,500}{200} = 17.5\%$$

*(Measurements must be in the same unit)*

**To express a % increase or decrease in change in quantity, put the change over the original and multiply by 100**

**Example** – Calculate the % increase of a class that went from 20 students to 25

$$\frac{5}{20} \times 100 = \frac{500}{20} = 25\%$$

*(change in quantity is 25 – 20 = 5)*

**To calculate the % Profit/ Loss put the profit/loss over the cost price and multiply by 100**

**Example** – A fruit importer buys apples for 30c and sells them for 42c. What is his percentage profit?

Profit = Sales – Costs = 42c – 30c = 12c *His profit*

$$\% \text{ profit} = \frac{\text{PROFIT}}{\text{COSTPRICE}} \times 100 = \frac{12}{30} \times 100 = \frac{1200}{30} = 40\%$$

*Answer*

**Example** – Fred sold his car for €750 making a profit of 25%. How much did he pay for the car. (This can also be asked as a VAT question and would be solved the very same way)

**Example** – Fred bought a fridge for €750 including VAT of 25%. What was the price before VAT was added.

Cost Price + Profit = 750

$$100\% + 25\% = 750$$

$$125\% = 750$$

$$1\% = \frac{750}{125} = 6$$

$$100\% = 6 \times 100 = \text{€ } 600$$

*750 is made up of Cost price plus the profit he made.*

*Cost is obviously 100% of cost, Profit is a further 25%*

*So the sale price was 125% of the cost price*

*Divide by 125 to find 1% of the cost price*

*Multiply by 100 to find cost price.*

**The car cost €600 or the fridge was €600 before VAT was added.**

## Currency -

The easiest way to solve currency questions is to use cross multiplication.

**Example** – A supplier buys 300 parts for \$336 each. They will be sold for a total of €138,000. Calculate the percentage profit on the cost price if the exchange rate is €1 = \$0.84.

$$300 \times \$336 = \$100,800$$

*Firstly work out total cost in dollars*

$$€1 = \$0.84$$

$$x = \$100,800$$

$$0.84x = 1(100,800)$$

$$x = \frac{100,800}{0.84}$$

$$x = €120,000$$

*Write down exchange rate*

*Underneath put the amount you want to convert*

*Cross Multiply*

*Divide across by number next to x, 0.84*

*This is the cost in euro*

$$\text{Profit} = \text{Sales} - \text{Costs} = €138,000 - €120,000 = €18,000$$

$$\text{Percentage Profit} = \frac{\text{PROFIT}}{\text{COSTPRICE}} \times \frac{100}{1} = \frac{18,000}{120,000} \times \frac{100}{1} = \frac{1,800,000}{120,000} = 15\%$$

## Interest Rates –

Interest is the money we receive or pay for investing or borrowing.

We need to learn interest rate formula

$$I = \frac{PxTxR}{100}$$

where

I = Interest  
 P = Principal (the amount you invest/  
 borrow)  
 T = Time  
 R = Rate

**Simple Interest** – Interest only calculated once over the term, T of the investment.

**Compound Interest** – Interest is added at the end of every year to create a new principal for the next year.

**Example** – Calculate the difference between the simple and compound interest on an investment of €6,000 at 7% p.a (per annum) for 3 years.

$$I = \frac{PxTxR}{100} = \frac{6,000 \times 3 \times 7}{100} = \frac{126,000}{100} = \text{€}1,260 \quad \textit{Simple interest (put figures into formula)}$$

**Compound Interest** (we calculate one years interest at a time and add this to principal)

		€
Year 1	Principal	6000
	Interest at 7%	<u>420</u>
Year 2	Principal	6420
	Interest at 7%	<u>449.40</u>
Year 3	Principal	6869.40
	Interest at 7%	<u>480.86</u>
Value of investment		7349.40

$$\text{Total interest} = (420 + 449.40 + 480.86) = 1349.40$$

$$\text{Difference between simple and compound} = 1349.40 - 1260 = 89.40$$

**Example** – What some of money invested at 6% p.a over 2 years would amount to €5,056.20.

To do this we first see what €100 would amount to at the same rate and time period

		€
Year 1	Principal	100
	Interest at 6%	<u>6</u>
Year 2	Principal	106
	Interest at 6%	<u>6.36</u>
		112.36

$$\begin{aligned} \text{€}100 &\longrightarrow \text{€}112.36 \\ x &\longrightarrow \text{€}5056.20 \end{aligned}$$

*If we invest €100 at 6% for 2 years we get €112.36 so we can cross multiply to investigate what we need to invest to get €5056.20*

$$\begin{aligned} 112.36x &= 100(5056.20) \\ 112.36x &= 505,620 \end{aligned}$$

*Cross Multiply  
Simplify*

$$x = \frac{505620}{112.36} = \text{€}4500$$

*Divide across by number next to x, 112.36.*

So if we invest **€4500** for 2 years at 6% we will get €5056.20

## Income Tax –

Questions normally ask you to calculate a person's net pay.

**Net pay (take home pay) = Gross Wages/ Income/ Salary – Tax Due**

**Tax Due = Gross Tax – Tax Credits**

To calculate the tax payable multiply the gross pay by the tax rate (sometimes there will be two tax rates).

**Example** – James has income of €30,000. Tax is charged on the first €14,000 at rate of 22%. The rest is charged at 40%. His Tax credits are €3,000. Calculate James' take home pay.

	€	
Income is 30,000	<b>30,000</b>	
14, 000 will be charged at 22%	=14 000 x 22% = 3080	
The remaining 16, 000 will be charged at 40%	=16 000 x 40% = <u>6400</u>	+
Gross Tax	= 9480	
Tax Credits	= <u>3000</u>	-
Tax Due	= <b>6480</b>	
Take home pay (€30,000 – €6,480)		= <b>€23,520</b>

**Example** – Joe has income of €30,000. Tax is charged on the first €14,500 at rate of 22%. The rest is charged at r%. His Tax credits are €3,200. Joe's take home pay is €23,655.

*In this example we must work backwards.*

*Firstly this is a two-interest rate question.*

*€14,500 is charged at 22%*

*The remainder (€30,000 – €14,500) = €15, 500 is charged at r%*

$$€30,000 - €23,655 = €6,345$$

$$€3,200 + €6,345 = €9,545$$

*Subtract the **take home** from the **income** to get the **Tax Due***

*Add the **Tax Due** to the **Tax Credits** to get the **Gross Tax***

Calculate 22% of €14,500

$$\frac{22}{100} \times 14,500 = €3,190$$

*This gives us the part of the Gross Tax which relates to the first €14,500*

$$€9,545 - €3,190 = €6,355$$

*This is the remainder of the Gross Tax which relates to the €15,500*

*So what we want to know is what % is this of €15,500.*

$$r = \frac{6,355}{15,500} \times 100 = 41\%$$

## Speed, Time and Distance -

For questions involving Speed, Time and Distance we need to learn the following formulae.

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}} \qquad \text{Time} = \frac{\text{Distance}}{\text{Speed}} \qquad \text{Distance} = \text{Speed} \times \text{Time}$$

Give your answer in kilometres per hour (km/h) unless otherwise asked.

If your answer is in kilometres per minute (km/m) multiply by 60 to get km/h.

**Example** – A journey of 9km took 20 mins. Find the average speed.

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

*Write down formula*

$$= \frac{9}{20} = 0.45 \text{ km/m}$$

*Answer here is in kilometres per minute*

$$0.45 \times 60 = \mathbf{27 \text{ km/h}}$$

*Multiply by 60 to get km/h*

**Example** – A bus leaves Mullingar at 11.05 and travels to Dublin 50 kms away, at an average speed of 20km/h. At what time does the bus arrive in Dublin.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

*Write down formula*

$$\text{Time} = \frac{50}{20} = 2.5 \text{ hours}$$

*Enter figures to give time in hours*

$$= 2.5 \times 60 = 150 \text{ minutes}$$

*Multiply by 60 to get minutes*

$$= 2 \text{ hours } 30 \text{ mins}$$

*Turn into Hours and Minutes*

$$11.05 +$$

*Add 2hrs 30mins to 11.05*

$$\underline{2.30}$$

$$\underline{13.35}$$

*Bus arrives at 13.35*

**Example** – At what average speed should I drive to cover 266km in 2 hours 20 mins.

$$2 \text{ hours } 20 \text{ mins} =$$

$$(2 \times 60) + 20 = 120 + 20 = 140 \text{ mins}$$

*We cannot use combinations of hrs and mins so change 2hrs 20mins into 140 minutes.*

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

*Write down formula*

$$= \frac{266}{140} = 1.9 \text{ km/ minute}$$

*Answer here is in kilometres per minute*

$$1.9 \times 60 = \mathbf{114 \text{ km/h}}$$

*Multiply by 60 to get km/h*

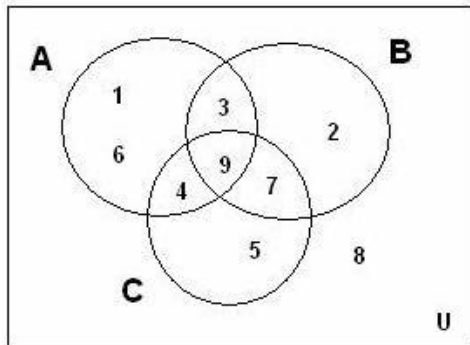
## Sets -

$A \cap B$	A intersection B – What is common to both
$A \cup B$	A union B – List all the elements in A and all the elements in B
$A'$	A complement – List everything outside of A
$A/B$	A without B – List the elements in A but don't include any of B
$\#A$	The cardinal number of A – How many elements are in A
$A \subset B$	A is a subset of B – Everything in A is also in B. Doesn't mean that everything in B is in A also.

These symbols can be combined:

$(A \cap B)'$	Everything outside of A intersection B
$(A \cup B) \setminus C$	All of A and all of B but don't list any elements of C

## Example -

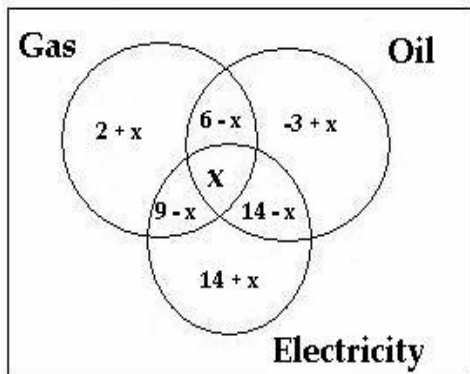


### Examples of things they may ask

$U = \{1,2,3,4,5,6,7,8,9\}$	
$A = \{1,3,4,6,9\}$	$\#A = 5$
$B = \{2,3,7,9\}$	$\#B = 4$
$C = \{4,5,7,9\}$	$\#C = 4$
$A \cap B = \{3,9\}$	$(A \cap B)' \cap C = \{4,5,7\}$
$A \cap B \cap C = \{9\}$	$(A \cup B) \setminus C = \{1,2,3,6\}$
$B \cap C = \{7,9\}$	$(A \cap B) \cap C' = \{3\}$
$A' = \{2,5,7,8\}$	$(A \setminus C) \cap (C \setminus B) = \{\}$
$A \setminus B = \{1,4,6\}$	$A' \cup (B \setminus C) = \{2,3,5,7,8\}$

**Example** – A survey of 50 households were asked whether they used gas, oil or electricity to heat their homes.

37 have electricity, 17 have gas, 19 have oil, 9 have gas and electricity, 14 have electricity and oil and 6 have gas and oil. X represents all 3. Find X.



X goes in center as it is in all 3

9 have G and E so  $(9 - x)$  in region shown  
 14 have E and O so  $(14 - x)$  in region shown  
 6 have G and O so  $(6 - x)$  in region shown

37 study Electricity. We already have  $x$  in there,  $(9 - x)$  in there and  $(14 - x)$  in there.

$$\begin{aligned} \text{The remainder is } & 37 - x - (14 - x) - (9 - x) \\ & = 14 + x \end{aligned}$$

which we put in the region shown and do likewise for Gas and Oil

Adding all the entries

$$(2 + x) + (6 - x) + (9 - x) + (14 + x) + (-3 + x) + (14 - x) + x = 50$$

$$42 + x = 50$$

$$x = 50 - 42 = 8$$

## Ratio and Proportion –

Firstly to find what fraction one number is of another we just put that number over the other.

Ratios describe the way that we divide things up.

If I want to divide some money between two people in the ratio 2:3 it means that for every €2 I give one person I give the other €3.

1:2:4 means that for every €1 A gets, B gets €2 and C gets €4

We can change a ratio by multiplying or dividing all the terms by the same number.

4:2 is the same as 2:1

*Divide both side by 2*

3:6:9 is the same as 1:2:3

*Divide all across by 3*

$1 : \frac{1}{2} : \frac{1}{4}$  is the same as 4:2:1

*Multiply across by 4 (always get rid of fractions when using ratios)*

To split amounts into ratios we do the following:

1. Add up the ratios to get the total number of shares
2. Divide the total amount to be split by the sum of the ratios to get the amount per share
3. Multiply each ratio by the amount per share.

**Example –** €350 is to be divided 2:5 between Tom and Frank. How much does each get?

**2:5**

$2+5 = 7$  shares

*Add the ratios to get the number of shares*

$\frac{350}{7} = 50$

*The amount each share will get*

Tom has 2 shares  $2 \times 50 = €100$

Frank has 5 shares  $5 \times 50 = €250$

**Example –** A sum of money was divided between Tom and Frank in the ratio 8:3. Tom received €200 more than Frank. How much did they each receive?

Difference between what each got was €200

*We are told this*

8 shares – 3 shares = €200

*The difference between the ratios*

5 shares = €300

*Simplify*

1 share =  $\frac{200}{5} = €40$

*Divide by 5 to find value of 1 share*

Tom has 8 shares  $8 \times 40 = €320$

*Multiply each ratio by the amount per share.*

Frank has 3 shares  $3 \times 40 = €120$

Total amount = €320 + €120 = €440

*Add to get total amount*



## Indices -

With questions involving indices we must break down all of the terms in the question to the same base number. We can use the rules below to simplify.

$$(a) 3^3 \cdot 3^4 = 3^{3+4} = 3^7 \quad (b) \frac{3^7}{3^3} = 3^{7-3} = 3^4 \quad (c) (3^2)^3 = 3^{2 \cdot 3} = 3^6 \quad (d) 3^0 = 1$$

$$(e) 9^{\frac{1}{2}} = \sqrt{9} = 3 \quad (f) 64^{\frac{1}{3}} = \sqrt[3]{64} = 4 \quad (g) 8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

$$(h) \frac{1}{3^5} = 3^{-5} \text{ and } 3^{-4} = \frac{1}{3^4}$$

### Example -

Solve for x in the equation

$$25^x = 5^{6-x}$$

$$(5^2)^x = 5^{6-x}$$

$$5^{2x} = 5^{6-x}$$

$$2x = 6-x$$

$$2x + x = 6$$

$$3x = 6$$

$$x = \frac{6}{3} = 2$$

*Change 25 into  $5^2$*

*Remove bracket to leave both sides in base 5*

*Let the indices (powers) equal each other  
x's to one side, numbers to the other*

*Divide across by 3*

### Example -

Simplify the equation and express in the form

$$\left(8^{\frac{1}{3}}\right) \left(4^{\frac{1}{4}}\right) = 2^{5-x}$$

$$\left(8^{\frac{1}{3}}\right) \left(4^{\frac{1}{4}}\right) = 2^{5-x}$$

$$(2^3)^{\frac{1}{3}} (2^2)^{\frac{1}{4}} = 2^{5-x}$$

$$(2^1)(2^{\frac{1}{2}}) = 2^{5-x}$$

$$2^{1\frac{1}{2}} = 2^{5-x}$$

$$1\frac{1}{2} = 5-x$$

$$\frac{3}{2} = 5-x$$

$$3 = 2(5-x)$$

$$3 = 10 - 2x$$

$$2x = 10 - 3$$

$$2x = 7$$

$$x = \frac{7}{2}$$

*The Equation*

*Turn everything into base 2*

*Multiply the powers*

$$(2^1)(2^{\frac{1}{2}}) = 2^{1\frac{1}{2}}$$

*Let the powers equal each other*

*Remove mixed fraction*

*Multiply across by 2*

*Multiply to remove bracket*

*x's to one side, numbers to the other*

*Divide across by 2*

## Surds –

**Surds are irrational numbers in the form  $\sqrt{\quad}$**

Some important points:

- $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$  and  $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$   
Therefore  $\sqrt{24}$  can be broken down into  $\sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$  therefore  $\sqrt{\frac{6}{4}} = \frac{\sqrt{6}}{\sqrt{4}} = \frac{\sqrt{6}}{2}$
- Terms with the same surd part can be added and subtracted  
Therefore  $5\sqrt{6} + 2\sqrt{6} - 3\sqrt{6} = 4\sqrt{6}$
- Any surd squared is equal to the term under the root sign

Therefore  $\sqrt{a^2} = a$   
 $\sqrt{5^2} = 5$

Equations involving surds can be solved by squaring both sides of the '='. This gets rid of the surd part to leave you with a simple or quadratic equation.

**Example –** Simplify  $(2 - 3\sqrt{2})(1 + 2\sqrt{2})$

$(2 - 3\sqrt{2})(1 + 2\sqrt{2})$	<i>The expression</i>
$= 2(1 + 2\sqrt{2}) - 3\sqrt{2}(1 + 2\sqrt{2})$	<i>Open up the brackets</i>
$= 2 + 4\sqrt{2} - 3\sqrt{2} - 2.3\sqrt{2}\sqrt{2}$	<i>Multiply in by number outside brackets</i>
$= 2 + \sqrt{2} - 6\sqrt{4}$	<i>Simplify</i>
$= 2 + \sqrt{2} - 6(2) = 2 + 2\sqrt{2} - 8$	<i>Simplify</i>
$= -6 + 2\sqrt{2}$	<i>Answer</i>

**Example –** Simplify  $\sqrt{3}(2\sqrt{6} - 4\sqrt{3}) - \sqrt{10}(3\sqrt{5} - 2\sqrt{10})$  and express your answer in the form  $a + b\sqrt{2}$

$\sqrt{3}(2\sqrt{6} - 4\sqrt{3}) - \sqrt{10}(3\sqrt{5} - 2\sqrt{10})$	<i>The expression</i>
$= 2\sqrt{3}\sqrt{6} - 4\sqrt{3}\sqrt{3} - 3\sqrt{10}\sqrt{5} + 2\sqrt{10}\sqrt{10}$	<i>Multiply in by number outside brackets</i>
$= 2\sqrt{18} - 4\sqrt{9} - 3\sqrt{50} + 2\sqrt{100}$	<i>Simplify using <math>\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}</math></i>
$= 2\sqrt{9}\sqrt{2} - 4(3) - 3\sqrt{25}\sqrt{2} + 2(10)$	<i>Change each term into either a number or <math>b\sqrt{2}</math></i>
$= 2(3)\sqrt{2} - 12 - 3(5)\sqrt{2} + 20$	<i>Simplify</i>
$= 6\sqrt{2} - 12 - 15\sqrt{2} + 20$	<i>Simplify</i>
$= 8 - 9\sqrt{2}$	<i>Answer in the form <math>a + b\sqrt{2}</math></i>

## Estimation -

The estimation question has normally two parts.

The first part will ask you to **estimate** an answer by rounding off numbers.

The second part will ask you to find the **exact** value of the same question.

It is important to take a step by step approach using the order of operation rules rather than entering all the information straight into the calculator.

**Brackets then powers then multiplication/ division then addition/ subtraction**

**Example** – By rounding to the nearest whole number, estimate the value of

$$\frac{131.5 - 1.73 \times \sqrt{0.64}}{35.4 - (5.1)^2}$$

$$\frac{131.5 - 1.73 \times \sqrt{0.64}}{35.4 - (5.1)^2}$$

*Our expression*

$$\frac{132 - 2 \times \sqrt{1}}{35 - (5)^2}$$

*Round off each figure to nearest whole number*

$$\frac{132 - 2 \times 1}{35 - 25}$$

*Work out Brackets and Powers*

$$\frac{132 - 2}{35 - 25}$$

*Must work out  $2 \times 1$  before  $132 - 2$ , simplify bottom*

$$\frac{130}{10} = 13$$

*Make sure you go through all the steps*

Then evaluate  $\frac{131.5 - 1.73 \times \sqrt{0.64}}{35.4 - (5.1)^2}$  correct to two decimal places.

$$\frac{131.5 - 1.73 \times \sqrt{0.64}}{35.4 - (5.1)^2}$$

*Our expression*

$$\frac{131.5 - 1.73 \times 0.8}{35.4 - 26.01}$$

*Work out Brackets and Powers*

$$\frac{131.5 - 1.384}{35.4 - 26.01}$$

*Work out  $1.73 \times 0.8$  first*

$$\frac{131.5 - 1.384}{35.4 - 26.01} = \frac{130.116}{9.39}$$

*Simplify*

$$\frac{130.116}{9.39} = 13.8568$$

*Simplify*

$$13.86$$

*Round off to two decimals*