## Area and Volume Revision Questions 1 of Paper 2

## Measurements -

Firstly make sure that before you put any measurements into a formula they are of the same unit. If the height is in mm and the radius is in cm then you will not have the correct answer.

If you are dealing with length give your answer in cm If you are dealing with area give your answer in $\mathrm{cm}^{2}$ If you are dealing with volume give your answer in $\mathrm{cm}^{3}$

Remember that $\pi$ is equal to 3.14 or $\frac{22}{7}$

> Be careful here $1 \mathrm{~cm}=10 \mathrm{~mm}$ BUT $\begin{aligned} & 1 \mathrm{~cm}^{3} \\ & \begin{array}{l}1 \mathrm{~cm}^{3} \\ \quad=10 \mathrm{~mm}^{3}\end{array} \\ & \quad=10 \mathrm{~cm} \times 1 \mathrm{~cm} \times 1 \mathrm{~cm} \\ & \quad=1000 \mathrm{~mm}^{3}\end{aligned}$

If you are asked to give your answer in terms of $\pi$ though do NOT put in 3.14 just leave $\pi$ in the answer.
For example a circle with radius 3 would have an area of $\pi r^{2}$ which we would write as $\pi(3)^{2}=9 \pi \mathrm{~cm}^{2}$
If they do not ask to express in terms of $\pi$ then we multiply in 3.14
$\pi(3)^{2}=(3.14)(9)=28.26 \mathrm{~cm}^{2}$

## Pythagoras -

In a right angle triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
Basically the long side squared is equal to the sum of the other sides squared.
This can be asked a number of ways but one of the most important and least obvious is when dealing with cones.

Example - Calculate the length of the missing side in the following example.


If we are given a cone we are sometimes given the radius and length but NOT the height. The volume of a cone formula requires the height not the length and students often make the mistake of putting the length in instead.
In this case we use Pythagoras to find the height and then put the height into the formula for $\mathbf{h}$. LENGTH IS NOT THE SAME AS HEIGHT.

Example - Calculate the height of the following cone.


$$
\begin{aligned}
& \text { Use Pythagoras } \\
& 13^{2}=5^{2}+x^{2} \\
& 169=25+x^{2} \\
& 169-25=x^{2} \\
& 144=x^{2} \\
& x=12
\end{aligned}
$$

Area and Perimeter of Basic Shapes -


## Examples - Find the (i) area and (ii) perimeter of the following shapes.



8


| Area | $=l \times b$ |
| :--- | :--- |
|  | $=7 \times 4=28 \mathrm{~cm}^{2}$ |
| Perimeter | $=2 l+2 b$ |
|  | $=2(7)+2(4)=14+8=22 \mathrm{~cm}$ |

Area of Triangle $=\frac{1}{2} b h$

$$
=\frac{1}{2}(8)(6)=24 \mathrm{~cm}^{2}
$$

Area of Shape $=$ Area of Triangle + Area of Rectangle

$$
\begin{aligned}
& =\frac{1}{2} b h+l \times b \\
& =\frac{1}{2}(9)(3)+(9 \times 5) \\
& =\frac{1}{2}(9)(3)+(9 \times 5)=13.5+45=58.5 \mathrm{~cm}^{2}
\end{aligned}
$$

$$
\begin{aligned}
\text { Perimeter }(\text { Circumference }) & =2 \pi r \\
& =2(3.14)(8)=50.24 \mathrm{~cm} \\
& =\pi r^{2} \\
\text { Area of Circle } & =(3.14)(8)^{2} \\
& =3.14(64)=201 \mathrm{~cm}^{2}
\end{aligned}
$$

Area of Shaded region $=$ Area of Square - Area of Circle

$$
\begin{aligned}
& \text { Area of Square }=l \times b=14 \times 14=196 \mathrm{~cm}^{2} \\
& \text { Area of Circle }=\pi r^{2}=\left(\frac{22}{7}\right)(7)^{2}=154 \mathrm{~cm}^{2}
\end{aligned}
$$

Area of Shaded region $=196 \mathrm{~cm}^{2}-154 \mathrm{~cm}^{2}$

$$
=42 \mathrm{~cm}^{2}
$$

## Volume and Surface Area -

Questions regarding volume and surface area require the use of formula that are found in the tables. The first thing to do in these questions is to write down the relevant formula then put in any known numbers.


Example - Find the (i) volume and (ii) total surface area of the rectangular solid below.


$$
\begin{aligned}
\text { Volume } & =l \times w \times h \\
& =12 \times 2 \times 6 \\
& =144 \mathrm{~cm}^{3}
\end{aligned} \begin{aligned}
\text { Total Surface Area } & =2 l w+2 l h \times 2 w h \\
& =2(12)(2)+2(12)(6) \times 2(2)(6) \\
& =48+144+24 \\
& =216 \mathrm{~cm}^{2}
\end{aligned}
$$

Example - A rectangular solid of length 3 cm and height 4 cm has a volume of 84 cm .
Calculate its width.

Volume $=l \times w \times h=84$
$3 \times w \times 4=84$
$12 w=84$
$w=\frac{84}{12}$
$w=7 \mathrm{~cm}$

Write down formula and let it $=84$
Fill in known values
Simplify
Divide across by 12
Width of the solid

Example - Calculate the volume of a prism that has a perpendicular height of 5 cm , a base of 10 cm and a length of 15 cm .


$$
\begin{aligned}
\text { Volume } & =\frac{1}{2} b \times h \times l \\
& =\frac{1}{2} 10 \times 5 \times 15 \\
& =375 \mathrm{~cm}^{3}
\end{aligned}
$$

Be careful when doing questions involving cones. The surface area formulae require the length of the side of the cone whereas the volume formula requires the height.

If you are given either the height or the length and you are looking to find the other use Pythagoras theorem.


Curved surface area $=2 \pi r h$ Total surface area $=2 \pi r h+2 \pi r^{2}$


Volume $=\frac{1}{3} \pi r^{2} h$
Curved surface area $=\pi r l$
Total surface area $=\pi r l+\pi r^{2}$

Example - Express to two decimal places the volume of a cone of radius 7 cm and height 10 cm .

Volume of cone $=\frac{1}{3} \pi r^{2} h \quad$ Write down the formula
$\frac{1}{3}(3.14)(7)^{2}(10) \quad$ Insert known values
$\frac{1}{3}(3.14)(49)(10)$
Simplify
$\frac{1}{3}$ (1538.6)
Simplify
$512.87 \mathrm{~cm}^{2} \quad$ Expressed to 2 decimals
Example - Express in terms of $\pi$ the volume of a cone of radius 5 cm and height 12 cm .
Volume of cone $=\frac{1}{3} \pi r^{2} h \quad$ Write down the formula
$=\frac{1}{3} \pi(5)^{2}(12) \quad$ Insert known values
$=\frac{1}{3} \pi(25)(12) \quad$ Simplify
$=100 \pi \quad$ Expressed in $\pi$
Example - The volume of a cone of radius 3 cm is $132 \mathrm{~cm}^{2}$. Find the (i) cone's height and hence (ii) the cone's total surface area.
(i) Volume of cone $=\frac{1}{3} \pi r^{2} h=132 \quad$ Write down the formula and let equal 132
$\frac{1}{3}(3.14)(3)^{2} h=132$
$\frac{1}{3}(3.14)(9) h=132$
Insert known values
Simplify
$9.42 h=132$
$h=\frac{132}{9.42}=14 \mathrm{~cm}$

## Simplify

Divide across by 9.42 and simplify
(ii) Before we can use the Total Surface Area formula we must find the slant length of the cone.
Use Pythagoras
If the height is 14 and the radius is 3 then:
$l^{2}=14^{2}+3^{2}$
$l^{2}=196+9$
$l^{2}=205$
$l=14.32$
Total Surface Area $=\pi r l+\pi r^{2}$
$=\pi(3)(14.32)+\pi(3)^{2}$
$=43 \pi+9 \pi$
$=52 \pi$

Write down the formula Insert known values


Simplify
Expressed in terms of $\pi$

Example - Express to two decimal places the volume of a cylinder of radius 14 cm and height 10 cm .

Volume of cylinder $=\pi r^{2} h$
$=(3.14)(14)^{2}(10)$
$=(3.14)(196)(10)$
$=6154.40 \mathrm{~cm}^{2}$

Write down the formula
Insert known values
Simplify
Expressed to 2 decimals

Example - Express in terms of $\pi$ the (i) volume and (ii) total surface area of a cylinder of radius 5 cm and height 12 cm .

| (i) Volume of Cylinder $=\pi r^{2} h$ |  | (ii) Total Surface Area $=2 \pi r h+2 \pi r^{2}$ |  |
| :---: | :---: | :---: | :---: |
| $=\pi(5)^{2}(12)$ | Insert values | $=2 \pi(5)(12)+2 \pi(5)^{2}$ | Insert value |
| $=\pi(25)(12)$ | Simplify | $=120 \pi+50 \pi$ | Simplify |
| $=300 \pi$ | Expressed in $\pi$ | $=170 \pi$ | Expressed |

Example - The volume of a cylinder of height 7 cm is $88 \mathrm{~cm}^{2}$. Find the cylinder's radius.
(i) Volume of cylinder $=\pi r^{2} h=88$
$\frac{22}{7} r^{2}(7)=88$
$22 r^{2}=88$
$r^{2}=\frac{88}{22}$
$r^{2}=4$
$r=2 \mathrm{~cm}$

Write down formula and let equal 88
Insert known values
Simplify
Divide across by 22
Simplify
Get the square root

The formulae for hemispheres are not in the logs tables but can be arrived at using the formulae for the sphere. A hemisphere is basically just half a sphere so for the volume and curved surface just get the sphere formula and divide by 2 .


Curved surface area $=4 \pi r^{2}$

Hemisphere
(not in $\log$ tables)
but a hemisphere is just half a sphere


Curved surface area $=2 \pi r^{2}$
Total surface area $=3 \pi r^{2}$

Example - Express to two decimal places the volume of a sphere of radius 6 cm .
Volume of sphere $=\frac{4}{3} \pi r^{3}$
Write down the formula
$=\frac{4}{3}(3.14)(6)^{3}$
Insert known values
$=\frac{4}{3}(3.14)(216)$

## Simplify

$=904.32 \mathrm{~cm}^{3}$

## Expressed to two decimals

Example - Express in terms of $\pi$ the curved surface area of a sphere of radius 14 cm .
Volume of sphere $=4 \pi r^{2} \quad$ Write down the formula
$=4\left(\frac{22}{7}\right)(14)^{2} \quad$ Insert known values
$=4\left(\frac{22}{7}\right)(196) \quad$ Simplify
$=\frac{17248}{7} \quad$ Simplify
$=2464 \mathrm{~cm}^{3} \quad$ Expressed in $\pi$
Example - The volume of a hemisphere is $57 \mathrm{~cm}^{2}$. Find the hemisphere's radius.
Volume of hemisphere $=\frac{2}{3} \pi r^{3}=57 \quad$ Write down the formula and let $=57$

$$
\frac{2}{3}(3.14) r^{3}=57
$$

## Insert known values

$2.09 r^{3}=57$

## Simplify

$r^{3}=\frac{57}{2.09}$
Divide across by 2.09
$r^{3}=27$
$r=3$
Simplify
Get the cubed root

## More complex problems -

Part (c) will involve more complex thinking, taking the formula from the tables and applying them to a two or three part question.
The first thing to always do is write down the shapes involved and their formulae. Also draw a rough sketch to help you visualise what is being asked.
It is also useful to leave $\pi$ and not put in 3.14 as a $\pi$ on each side of an equation will cancel each other out.
The following are the types of questions asked in section (c) and examples of each follow on the subsequent pages.

1. Recasting - one object is melted down and made into another
2. Displacement of water - one object placed into a water, the amount the water rises is equal to the volume of the object.
3. Shapes within a shape - calculate the area remaining when a number of small objects are placed in a large object.
4. Two shapes in one - find the volume of a shape that is made from two other shapes.
5. Flow of liquid - calculate the time taken for liquid to flow through a cylinder.

## Recasting -

For this question the volumes of the two shapes will be equal so begin by writing down the relevant formulae and letting them equal one another.
Fill in all the known numbers and then solve for whatever is left.
Example - A cylinder and a sphere have equal volumes. If the radius of the sphere is 6 cm and the radius of the cylinder is 8 cm , calculate the height of the cylinder.

Volume of cylinder $=\pi r^{2} h \quad$ Write down formula
Volume of sphere $=\frac{4}{3} \pi r^{3}$
Write down formula
$\pi r^{2} h=\frac{4}{3} \pi r^{3}$
Let formula equal each other
$\pi(8)^{2} h=\frac{4}{3} \pi(6)^{3}$
Insert known values
$64 h=\frac{4}{3}(216)$
Cancel $\pi$ on both sides and simplify
$64 h=288$

## Simplify

$h=\frac{288}{64}$
Divide across by 64
$=4.5 \mathrm{~cm}$
Height of cylinder

## Displacement of water -

An object or group of objects is placed into water. The volume of the water that rises is equal to the volume of the object or objects placed in the water.

Example - When a solid sphere is dropped into a cylinder partly filled with water, the level of the water rises by hcm . If the diameter of the cylinder is 16 cm , calculate h .

Draw a rough sketch


Write down the volume of the sphere and the volume of water displaced. Put down the formulae and enter known numbers.
Volume of Sphere $=\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi(6)^{3}=\frac{4}{3} \pi 216=288 \pi \quad($ sphere $)$
Volume of Water displaced $=\pi r^{2} h=\pi(8)^{2} h=64 \pi h \quad$ (cylinder)
(Remember if diameter 16 cm then the radius is 8 cm )
Volume of Water displaced $=$ Volume of Sphere
$\begin{array}{ll}\text { 64 } \pi \text { h }=288 \pi & \text { Water displaced }=\text { volume of object } \\ 64 h=288 & \text { We can cancel } \pi \text { on both sides }\end{array}$
$64 h=288 \quad$ We can cancel $\pi$ on both sides
$h=\frac{288}{64}=4.5 \mathrm{~cm} \quad$ Divide across by 64 and simplify

## Shapes within a shape -

Small objects are placed in a larger object and we are asked to find the area or volume remaining.

Example - Three tennis balls of diameter 7 cm are placed inside a cylindrical container. The diameter of the cylinder is equal to the diameter of the tennis ball and the height is such that the tennis balls cannot move about. What fraction of space is occupied by the tennis balls?

Draw a rough sketch


Write down the volume of the large object, in this case a cylinder. The diameter is 14 cm so the radius is 7 cm .
The height is 3 tennis balls therefore $3 \times 14=42 \mathrm{~cm}$.
Volume of cylinder $=\pi r^{2} h=\pi(7)^{2} 42=2058 \pi$
Next find the volume of one tennis ball, radius 7 cm .
Volume of sphere $=\frac{4}{3} \pi(7)^{3}=\frac{4}{3} \pi(343)=457 \frac{1}{3} \pi$
Total volume of 3 tennis balls $=3 \times 457 \frac{1}{3} \pi=1372 \pi$
Space occupied by tennis balls $=\frac{1372 \pi}{2058 \pi}=\frac{1372}{2058}=\frac{2}{3}$

## Two shapes in one -

Sometimes we will be asked to find the volume of a shape that consists of a combination of cones, cylinders, spheres etc.

In this case just add the volume of each shape separately to find the total volume of the figure.

Example - Calculate the volume of the shapes below.


$$
\begin{aligned}
\text { Shape } & =\text { Cone }+ \text { Cylinder } \\
& =\frac{1}{3} \pi r^{2} h+\pi r^{2} h \\
& =\frac{1}{3} \pi(4)^{2}(8)+\pi(4)^{2} 10 \\
& =\frac{1}{3} \pi(16)(8)+\pi(16) 10 \\
& =42 \frac{2}{3} \pi+160 \pi \\
& =202 \frac{2}{3} \pi c m^{3}
\end{aligned}
$$



$$
\begin{aligned}
\text { Shape } & =\text { Cone }+ \text { Hemisphere } \\
& =\frac{1}{3} \pi r^{2} h+\frac{4}{3} \pi r^{3} \\
& =\frac{1}{3} \pi(4)^{2}(10)+\frac{4}{3} \pi(4)^{3} \\
& =\frac{1}{3} \pi(16)(10)+\frac{4}{3} \pi(64) \\
& =53 \frac{1}{3} \pi+85 \frac{1}{3} \pi \\
& =138 \frac{2}{3} \pi \mathrm{~cm}^{3}
\end{aligned}
$$



$$
\begin{aligned}
\text { Shape } & =\text { Prism }+ \text { Rectangular Solid } \\
& =\frac{1}{2} l \times w \times h+l \times w \times h \\
& =\frac{1}{2}(12 \times 8 \times 2)+(12 \times 8 \times 5) \\
& =96+480 \\
& =576 \mathrm{~cm}^{3}
\end{aligned}
$$

## Flow of liquid -

Example - Water flows through a circular pipe of radius 2 cm at the rate of $7 \mathrm{~cm} / \mathrm{sec}$ into a rectangular tank that measures 1.2 m long by 1.1 m wide by 30 cm high. How long will it take to fill the tank?

## ANSWER -

Time taken to fill the tank $=\frac{\text { Size of the TANK }}{\text { Rate of flow }}$

We need to find out two things first.

1. The size of the tank
2. The rate of flow

First change all our measurements into cm
$1.2 \mathrm{~m}=120 \mathrm{~cm}$
$1.1 \mathrm{~m}=110 \mathrm{~cm}$
$30 \mathrm{~cm}=30 \mathrm{~cm}$
Size of tank $=$ Rectangular Solid $=l \times w \times h$

$$
=120 \times 110 \times 30=396000 \mathrm{~cm}^{3}
$$

To find the rate of flow we need to find the volume of the cylinder.
Speed of $7 \mathrm{~cm} /$ s means that the height of the cylinder is 7 cm
Volume of cylinder $=\pi r^{2} h$

$$
=\left(\frac{22}{7}\right)(2)^{2}(7)=88 \mathrm{~cm}^{3}
$$

Therefore 88 cubic cm passes through into the tank per second.
$\frac{\text { Size of the TANK }}{\text { Rate of flow }}=\frac{396000}{88}=4500$ seconds

| $\frac{4500}{60}=75 \mathrm{mins}$ | Divide across by 60 to get hours (60 secs in minute) |
| :--- | :--- |
| $\frac{75}{60}=1.25 \mathrm{hrs}$ | Divide across by 60 to get hours ( 60 mins in hour) |
| 1 hour 15 mins | The time in hours and minutes |

