Area and Volume Revision -Questions 1 of Paper 2

Measurements -

Firstly make sure that before you put any measurements into a formula they are of the same unit. If the height is in mm and the radius is in cm then you will not have the correct answer.

If you are dealing with **length** give your answer in cmIf you are dealing with **area** give your answer in cm^2 If you are dealing with **volume** give your answer in cm^3

Remember that π is equal to 3.14 or $\frac{22}{7}$

Be careful here 1 cm = 10 mm BUT $1 \text{ cm}^3 \neq 10 \text{ mm}^3$ $1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$ $= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$ $= 1000 \text{ mm}^3$

If you are asked to give your answer in terms of π though do NOT put in 3.14 just leave π in the answer.

For example a circle with radius 3 would have an area of πr^2 which we would write as $\pi(3)^2 = 9\pi \ cm^2$

If they do not ask to express in terms of π then we multiply in 3.14 $\pi(3)^2 = (3.14)(9) = 28.26 \ cm^2$

Pythagoras -

In a right angle triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Basically the long side squared is equal to the sum of the other sides squared.

This can be asked a number of ways but one of the most important and least obvious is when dealing with cones.

Example – Calculate the length of the missing side in the following example.

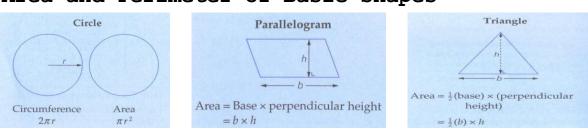
x 4	$x^{2} = 4^{2} + 3^{2}$ $x^{2} = 16 + 9$ $x^{2} = 25$ $x = 5$
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If we are given a cone we are sometimes given the radius and length but NOT the height. The volume of a cone formula requires the height not the length and students often make the mistake of putting the length in instead.

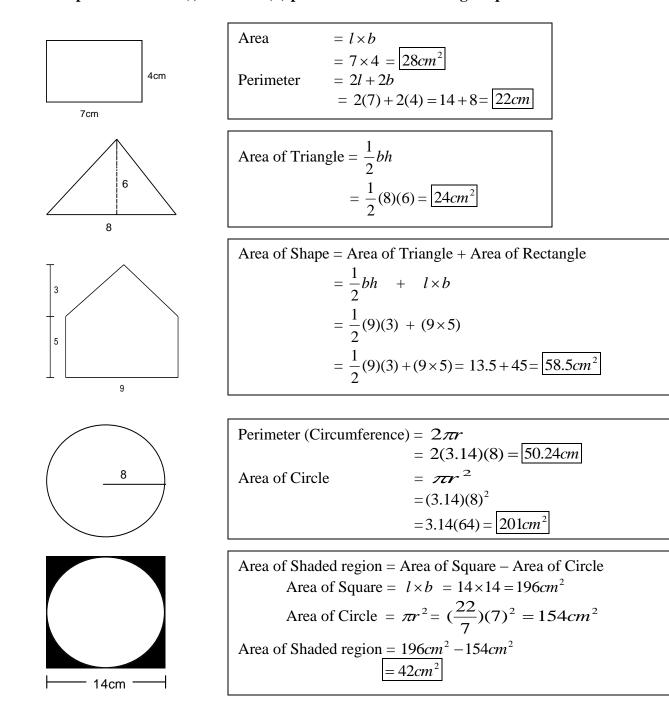
In this case we use Pythagoras to find the height and then put the height into the formula for **h.** LENGTH IS NOT THE SAME AS HEIGHT.

Example – Calculate the height of the following cone.





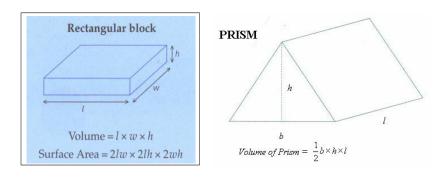
Examples – Find the (i) area and (ii) perimeter of the following shapes.



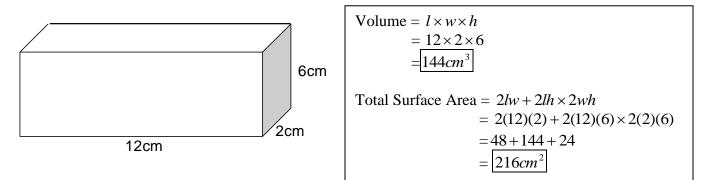
Area and Perimeter of Basic Shapes

Volume and Surface Area -

Questions regarding volume and surface area require the use of formula that are found in the tables. The first thing to do in these questions is to write down the relevant formula then put in any known numbers.



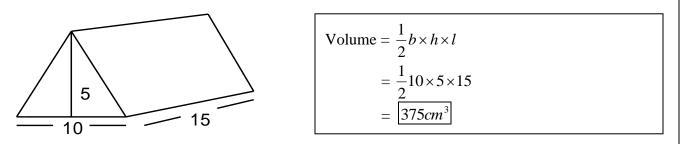
Example – Find the (i) volume and (ii) total surface area of the rectangular solid below.



Example – A rectangular solid of length 3cm and height 4 cm has a volume of 84cm. Calculate its width.

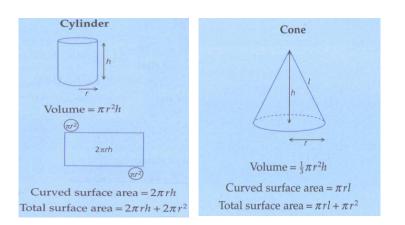
Volume = $l \times w \times h = 84$	Write down formula and let $it = 84$
$3 \times w \times 4 = 84$	Fill in known values
12w = 84	Simplify
$w = \frac{84}{12}$	Divide across by 12
w = 7cm	Width of the solid

Example – Calculate the volume of a prism that has a perpendicular height of 5cm, a base of 10cm and a length of 15cm.



Be careful when doing questions involving cones. The surface area formulae require the length of the side of the cone whereas the volume formula requires the height.

If you are given either the height or the length and you are looking to find the other use Pythagoras theorem.



Example – Express to two decimal places the volume of a cone of radius 7cm and height 10 cm.

Volume of cone = $\frac{1}{3}\pi r^2 h$	Write down the formula
$\frac{1}{3}(3.14)(7)^2(10)$	Insert known values
$\frac{1}{3}(3.14)(49)(10)$	Simplify
$\frac{1}{3}$ (1538.6)	Simplify
$512.87cm^2$	Expressed to 2 decimals

Example – Express in terms of π the volume of a cone of radius 5cm and height 12cm.

Volume of cone = $\frac{1}{3}\pi r^2 h$	Write down the formula
$=\frac{1}{3}\pi(5)^2(12)$	Insert known values
$=\frac{1}{3}\pi(25)(12)$	Simplify
$=$ 100 π	Expressed in π

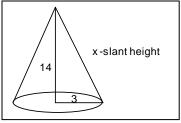
Example – The volume of a cone of radius 3 cm is 132 cm^2 . Find the (i) cone's height and hence (ii) the cone's total surface area.

(i) Volume of cone = $\frac{1}{3}\pi r^2 h = 132$	Write down the formula and let equal 132
$\frac{1}{3}(3.14)(3)^2h = 132$	Insert known values
$\frac{1}{3}(3.14)(9)h = 132$	Simplify
9.42h = 132	Simplify
$h = \frac{132}{9.42} = 14cm$	Divide across by 9.42 and simplify

(ii) Before we can use the Total Surface Area formula we must find the slant **length** of the cone.

Use Pythagoras If the height is 14 and the radius is 3 then: $l^{2} = 14^{2} + 3^{2}$ $l^{2} = 196 + 9$ $l^{2} = 205$ l = 14.32

Total Surface Area = $\pi r l + \pi r^2$ = $\pi(3)(14.32) + \pi(3)^2$ = $43\pi + 9\pi$ = 52π



Write down the formula Insert known values Simplify Expressed in terms of π

Example – Express to two decimal places the volume of a cylinder of radius 14cm and height 10 cm.

Volume of cylinder = $\pi r^2 h$	Write down the formula
$=(3.14)(14)^2(10)$	Insert known values
=(3.14)(196)(10)	Simplify
$= 6154.40 cm^2$	Expressed to 2 decimals

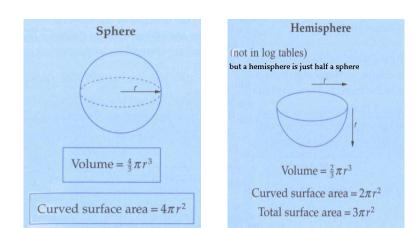
Example – Express in terms of π the (i) volume and (ii) total surface area of a cylinder of radius 5cm and height 12cm.

(i) Volume of Cylinder = $\pi r^2 h$	(ii) Total Surface Area = $2\pi rh + 2\pi r^2$
$= \pi(5)^2(12)$ Insert values $= \pi(25)(12)$ Simplify $= 300\pi$ Expressed in π	$= 2\pi(5)(12) + 2\pi(5)^{2}$ Insert values $= 120\pi + 50\pi$ Simplify $= 170\pi$ Expressed in π

Example – The volume of a cylinder of height 7cm is $88 cm^2$. Find the cylinder's radius.

(i) Volume of cylinder = $\pi r^2 h = 88$	Write down formula and let equal 88
$\frac{22}{7}r^2(7) = 88$	Insert known values
$22r^2 = 88$	Simplify
$r^2 = \frac{88}{22}$	Divide across by 22
$r^2 = 4$	Simplify
r = 2cm	Get the square root

The formulae for hemispheres are not in the logs tables but can be arrived at using the formulae for the sphere. A hemisphere is basically just half a sphere so for the volume and curved surface just get the sphere formula and divide by 2.



Example – Express to two decimal places the volume of a sphere of radius 6cm.

Volume of sphere $=\frac{4}{3}\pi r^3$	Write down the formula
$=\frac{4}{3}(3.14)(6)^3$	Insert known values
$=\frac{4}{3}(3.14)(216)$	Simplify
= 904.32 <i>cm</i> ³	Expressed to two decimals

Example – Express in terms of π the curved surface area of a sphere of radius 14cm.

Volume of sphere = $4\pi r^2$	Write down the formula
$=4(\frac{22}{7})(14)^2$	Insert known values
$=4(\frac{22}{7})(196)$	Simplify
$=\frac{17248}{7}$	Simplify
= 2464 <i>cm</i> ³	Expressed in π

Example – The volume of a hemisphere is $57 \, cm^2$. Find the hemisphere's radius.

Volume of hemisphere =
$$\frac{2}{3}\pi r^3 = 57$$
 Writh

 $\frac{2}{3}(3.14)r^3 = 57$
 Inset

 $2.09r^3 = 57$
 Simple

 $r^3 = \frac{57}{2.09}$
 Dive

 $r^3 = 27$
 Simple

 $r = 3$
 Get

Write down the formula and let = 57 Insert known values Simplify Divide across by 2.09 Simplify Get the cubed root

More complex problems -

Part (c) will involve more complex thinking, taking the formula from the tables and applying them to a two or three part question.

The first thing to always do is write down the shapes involved and their formulae. Also draw a rough sketch to help you visualise what is being asked.

It is also useful to leave π and not put in 3.14 as a π on each side of an equation will cancel each other out.

The following are the types of questions asked in section (c) and examples of each follow on the subsequent pages.

- 1. Recasting one object is melted down and made into another
- 2. Displacement of water one object placed into a water, the amount the water rises is equal to the volume of the object.
- 3. Shapes within a shape calculate the area remaining when a number of small objects are placed in a large object.
- 4. Two shapes in one find the volume of a shape that is made from two other shapes.
- 5. Flow of liquid calculate the time taken for liquid to flow through a cylinder.

Recasting -

For this question the volumes of the two shapes will be equal so begin by writing down the relevant formulae and letting them equal one another.

Fill in all the known numbers and then solve for whatever is left.

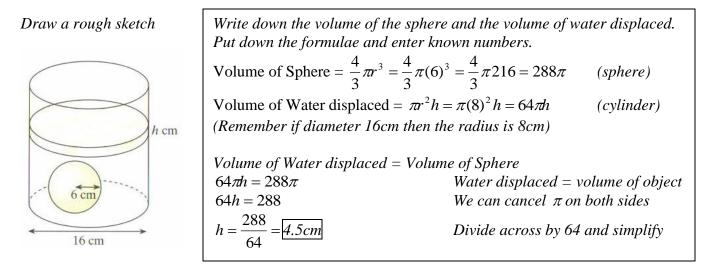
Example – A cylinder and a sphere have equal volumes. If the radius of the sphere is 6cm and the radius of the cylinder is 8cm, calculate the height of the cylinder.

Volume of cylinder = $\pi r^2 h$	Write down formula
Volume of sphere $=\frac{4}{3}\pi r^3$	Write down formula
$\pi r^2 h = \frac{4}{3} \pi r^3$	Let formula equal each other
$\pi(8)^2 h = \frac{4}{3}\pi(6)^3$	Insert known values
$64h = \frac{4}{3}(216)$	Cancel π on both sides and simplify
64 <i>h</i> = 288	Simplify
$h = \frac{288}{64}$	Divide across by 64
=4.5cm	Height of cylinder

Displacement of water -

An object or group of objects is placed into water. The volume of the water that rises is equal to the volume of the object or objects placed in the water.

Example – When a solid sphere is dropped into a cylinder partly filled with water, the level of the water rises by h cm. If the diameter of the cylinder is 16cm, calculate h.

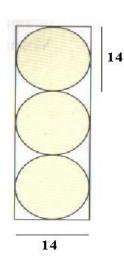


Shapes within a shape -

Small objects are placed in a larger object and we are asked to find the area or volume remaining.

Example – Three tennis balls of diameter 7cm are placed inside a cylindrical container. The diameter of the cylinder is equal to the diameter of the tennis ball and the height is such that the tennis balls cannot move about. What fraction of space is occupied by the tennis balls?

Draw a rough sketch



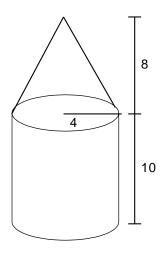
Write down the volume of the large object, in this case a cylinder. The diameter is 14cm so the radius is 7cm. The height is 3 tennis balls therefore $3 \times 14 = 42$ cm. Volume of cylinder $= \pi r^2 h = \pi (7)^2 42 = 2058\pi$ Next find the volume of one tennis ball, radius 7cm. Volume of sphere $= \frac{4}{3}\pi (7)^3 = \frac{4}{3}\pi (343) = 457\frac{1}{3}\pi$ Total volume of 3 tennis balls $= 3 \times 457\frac{1}{3}\pi = 1372\pi$ Space occupied by tennis balls $= \frac{1372\pi}{2058\pi} = \frac{1372}{2058} = \boxed{\frac{2}{3}}$

Two shapes in one -

Sometimes we will be asked to find the volume of a shape that consists of a combination of cones, cylinders, spheres etc.

In this case just add the volume of each shape separately to find the total volume of the figure.

Example – Calculate the volume of the shapes below.



Shape = Cone + Cylinder

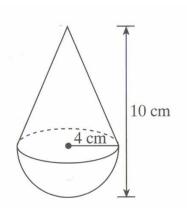
$$= \frac{1}{3}\pi r^{2}h + \pi r^{2}h$$

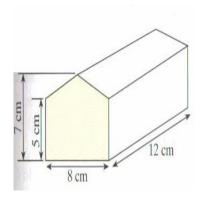
$$= \frac{1}{3}\pi (4)^{2}(8) + \pi (4)^{2}10$$

$$= \frac{1}{3}\pi (16)(8) + \pi (16)10$$

$$= 42\frac{2}{3}\pi + 160\pi$$

$$= \boxed{202\frac{2}{3}\pi cm^{3}}$$





Shape = Cone + Hemisphere

$$= \frac{1}{3}\pi r^{2}h + \frac{4}{3}\pi r^{3}$$

$$= \frac{1}{3}\pi (4)^{2}(10) + \frac{4}{3}\pi (4)^{3}$$

$$= \frac{1}{3}\pi (16)(10) + \frac{4}{3}\pi (64)$$

$$= 53\frac{1}{3}\pi + 85\frac{1}{3}\pi$$

$$= \boxed{138\frac{2}{3}\pi cm^{3}}$$

Shape = Prism + Rectangular Solid

$$= \frac{1}{2}l \times w \times h + l \times w \times h$$

$$= \frac{1}{2}(12 \times 8 \times 2) + (12 \times 8 \times 5)$$

$$= 96 + 480$$

$$= 576cm^{3}$$

Flow of liquid -

Example – Water flows through a circular pipe of radius 2cm at the rate of 7cm/ sec into a rectangular tank that measures 1.2m long by 1.1m wide by 30cm high. How long will it take to fill the tank?

ANSWER –

Time taken to fill the tank = $\frac{Size \ of \ the \ TANK}{Rate \ of \ flow}$

We need to find out two things first.

- 1. The size of the tank
- 2. The rate of flow

First change all our measurements into cm

1.2m = 120cm1.1m = 110cm30cm = 30cm

Size of tank = Rectangular Solid = $l \times w \times h$

 $= 120 \times 110 \times 30 = 396000 cm^3$

To find the rate of flow we need to find the volume of the cylinder. Speed of 7cm/ s means that the height of the cylinder is 7cm

Volume of cylinder = $\pi r^2 h$ = $(\frac{22}{7})(2)^2(7) = 88cm^3$

Therefore 88 cubic cm passes through into the tank per second.

 $\frac{Size of the TANK}{Rate of flow} = \frac{396000}{88} = 4500 \text{ seconds}$

$\frac{4500}{60} = 75 mins$	Divide across by 60 to get hours (60 secs in minute)
$\frac{75}{60} = 1.25 hrs$	Divide across by 60 to get hours (60 mins in hour)
1 hour 15 mins	The time in hours and minutes