Algebra, Functions and Graphs Revision Sheet – Questions 3,4,5 and 6 of Paper 1

Simple Equations –

Step 1 Get rid of brackets or fractions
Step 2 Take the x's to one side of the equals sign and the numbers to the other (remember to change the sign when crossing the '=')
Step 3 Divide across by the number next to the x.

Example – Solve for x,
3(2x - 1) = 4x

6x –3 = 4x	*Get rid of brackets by multiplying*
6x –4x = +3	*x's to one side, numbers to the other*
2x = 3	
$x = \frac{3}{2}$	*Divide across by 2*

Example – Solve for x,
$\frac{x-7}{2} = \frac{x+3}{6}$

6(x - 7) = 2(x + 3)	*Cross multiply to get rid of the fractions*
6x – 42 = 2x + 6	*Get rid of brackets by multiplying*
6x –2x = 6 + 42	*x's to one side, numbers to the other*
4x = 48	
$x = \frac{48}{4} = 12$	*Divide across by 4*

Substitution –

Write out the question again substituting numbers for letters.

Example – Find the value of $x^2 - 5xy$ when x = 3 and y = -2

$x^2 - 5xy$	*Write out expression*
$= (3)^2 - 5(3)(-2)$	*Substitute in numbers for x and y*
= 9 + 30	*Evaluate*
= 39	

Factorising –

Factorising basically means putting an expression into brackets.
There are 4 types of factorising and you need to be familiar with each.

1. Common factor – This involves taking a term that is common to all the terms in the expression outside of the brackets.

Example – Factorise $6x^2 - 18x$

$6x^2 - 18x = 6x(x-3)$ *Take out the common factor 6x*

2. Factors by Grouping – We use this method when there is 4 terms and no factor common to all of the terms. In this case we group terms in pairs which have a common factor.

Example – Factorise $x^2 - ab - bx + ax$

$x^2 - ab - bx + ax$	*The expression*
$x^2 - bx + ax - ab$	*Rearrange so we can take x out of first pair and a out of second*
$x(x-b) + a(x-b)$	*Take x out of first pair and a out of second.* ***Brackets should be same.***
$(x-b)(x+a)$	*Take out the common factor (x - b)*

3. Difference of two squares – We use this method when we have an expression in the form $a^2 - b^2$ which we can turn into (x + a)(x – a)

Examples **(i)** $x^2 - 25 = (x+5)(x-5)$

(ii) $4x^2 - 49 = (2x+7)(2x-7)$

Sometimes the terms being squared will be in brackets

(iii) $(x-4)^2 - (2x+2)^2 = ((x-4)+(2x-2))((x-4)-(2x+2))$
$= (x-4+2x-2)(x-4-2x-2) = (3x-6)(-x-6)$

Sometimes you may have to take out a common factor first.

(iv) $2x^2 - 18 = 2(x^2 - 9) = 2(x+3)(x-3)$

An expression can be changed into an equation by adding = 0 to the end
If there is an equals we are being asked to find values of x not just factorise.
In that case we factorise as above and then let each bracket = 0

$x^2 - 25 = 0$
$(x+5)(x-5) = 0$
$x + 5 = 0$ **AND** $x - 5 = 0$
$x = -5$ $x = 5$

$6x^2 - 18x = 0$
$6x(x-3) = 0$
$6x = 0$ **AND** $x - 3 = 0$
$x = 0$ $x = 3$

4. Factorising Quadratic Equations.

A quadratic expression is one in the form $ax^2 + bx + c$
Quadratics are a little bit harder to factorise and we use a trial and error approach.

Firstly all quadratics are factorised into two sets of brackets
$ax^2 + bx + c =$
( )( )

The first term in each bracket should multiply to give **a**
The second term in each bracket should multiply to give **c**
The product of the outer terms plus added to the product of the inner terms give **b**

Example - $x^2 + 7x + 6$

$(x+6)(x+1)$

The first term in each bracket should multiply to give x^2
$x \times x = x^2$
The second term in each bracket should multiply to give 6
$6 \times 1 = 6$ *2 and 3 also multiply to give 6 but wouldn't work below*
The product of the outer terms plus added to the product of the inner terms give 7x
$(x)(1) + (6)(x) = x + 6x = 7x$

Example - $2x^2 - 5x - 12$

$(2x+3)(x-4)$

The first term in each bracket should multiply to give $2x^2$
$2x \times x = 2x^2$
The second term in each bracket should multiply to give -12
$3 \times -4 = -12$ 6×2 *or* 12×1 *won't work below*
The product of the outer terms plus added to the product of the inner terms give 7x
$(2x)(-4) + (3)(x) = -8x + 3x = -5x$

If you cannot find the solutions (sometimes there are no even solutions) you can use the formula

$$\boldsymbol{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A worked example using the formula is on page 5

Expressing as a Single Fraction -

Step 1 – Find a lowest common denominator (usually all the bottom terms multiplied)
Step 2 – Multiply each top term by any terms NOT underneath it

$\frac{1}{x+1}+\frac{2}{x-3}$	*Common denominator is (x + 1)(x – 3)*
$\longrightarrow \frac{1(x-3)+2(x+1)}{(x+1)(x-3)}$	*Top terms times all terms NOT below them*
$\longrightarrow \frac{x-3+2x+2}{(x+1)(x-3)}$	*Multiply out to simplify*
$\longrightarrow \frac{3x-1}{(x+1)(x-3)}$	Answer

Equations with Algebraic Fractions –

These questions involve solving equations that have fractions and require you to get rid of the fractions before you can solve the equation.

Step 1 – Find a lowest common denominator (usually all the bottom terms multiplied)
Step 2 – Multiply each top term by any terms NOT underneath it
Step 3 – Remove the common denominator
Step 4 – Bring all the terms to the left of the '=' and simplify
Step 5 – Solve the quadratic equation by factorising (if possible) and letting each bracket equal 0. If it cannot be factorised you must use the formula (see next page).

Example Solve $\frac{1}{x+1}+\frac{2}{x-3}=3$

$\frac{1}{x+1}+\frac{2}{x-3}=\frac{3}{1}$	*Common denominator is (x + 1)(x – 3)*
$\longrightarrow \frac{1(x-3)(1)+2(x+1)(1)=3(x+1)(x-3)}{(x+1)(x-3)}$	*Top terms times all terms NOT below them*
$\longrightarrow$ 1(x – 3) + 2(x +1) = 3{(x +1)(x – 3)}	*Remove denominator (bottom section)*
$\longrightarrow$ x – 3 + 2x + 2 = 3{x(x+1) +1(x – 3)}	*Multiply out, open (x + 1)(x – 3)*
$\longrightarrow$ x – 3 + 2x + 2 = 3{ $x^2+x+x-3$ }	*Simplify*
$\longrightarrow$ x – 3 + 2x + 2 = 3{ x^2+2x-3 }	*Simplify*
$\longrightarrow$ 3x - 1 = $3x^2+6x-9$	*Simplify*
$\longrightarrow -3x^2-6x+9+3x-1=0$	*Everything to one side.*
$\longrightarrow -3x^2-3x+8=0$	*If the equation can be factorised do so and let each bracket equal 0. If not use the FORMULA.*

Turn over to see how the FORMULA can be used to find our x values. The formula MUST be learned off by heart.

the ***FORMULA***

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula will give us the two roots (x values) for any quadratic equation. Where possible however it is easier to factorise and let each bracket equal 0.

$-3x^2 - 3x + 8 = 0$	*A quadratic that cannot be factorised.*
$a = -3 \;\; b = -3 \;\; c = 8$	*The values of a, b and c for the formula.*
$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	*The formula*
$\dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(-3)(8)}}{2(-3)}$	*Substitute in the a, b and c values.*
$\dfrac{+3 \pm \sqrt{9 \;\; +96}}{-6}$	*Simplify*
$\dfrac{3 \pm \sqrt{104}}{-6}$	*Simplify*
$\dfrac{3 \pm 10.2}{-6}$	*Remove square root*
$x = \dfrac{3+10.2}{-6}$ *and* $x = \dfrac{3-10.2}{-6}$	*Split into the + and – parts*
$x = \dfrac{13.2}{-6}$ *and* $x = \dfrac{-7.2}{-6}$	*Simplify*
x = - 2.2 and x = 1.2	*The roots of the equation.*

Sometimes we will have to use our x values to find out something to solve a **similar** equation. To do this we let the x values equal what is replacing x in the **similar** equation.

Example – Solve for $-3x^2 - 3x + 8 = 0$

Hence or otherwise solve $-3(t-1)^2 - 3(t-1) + 8 = 0$

Solve the equation using formula as above to get
x = - 2.2 and x = 1.2

The two equations above are the same except that we use x in one and (t – 1) in the other. Therefore comparing both equations we can say that t – 1 = x

Since we have two values for x let each of these equal t – 1 to get the t values

t – 1 = - 2.2	*AND*	*t – 1 = 1.2*
t = -2.2 + 1		*t = 1.2 + 2*
t = -1.2		*t = 2.2*

Simultaneous Equations –

This will be a question where we are looking to find the value of TWO unknowns. We must form equations from written problems using x and y to represent the unknown values.
We then use these equations to solve for x and y, find out their values.
It is important when writing out both equations that we arrange as follows:
x term + or – y term = number

Example – A person has €650 made up of €5 and €10 notes. There are 87 notes in total. Taking x to be the number of €5 notes and y to be the number of €10 notes, write down two equations in x and y to represent this information.

Equation 1
The number of x notes and number of y notes gives total of 87
$x + y = 87$

Equation 2
The value of the €5's plus the value of the €10 gives a total value of €650
$5x + 10y = 650$

$x + y = 87$ $5x + 10y = 650$	**Equation 1** **Equation 2**	*We write one equation directly above the other.*
$5x + 5y = 435$ $5x + 10y = 650$	**Equation 1** **Equation 2**	*We multiply the top equation by 5 to get the x values the same.*
$-\not{5x} - 5y = -435$ $\not{5x} + 10y = 650$ $5y = 115$	**Equation 1** **Equation 2**	*To cancel the x's their signs must be different so we change ALL the signs in Equation 1* *We then add or subtract the y's* $-5y + 10y = 5y$ *We add or subtract the numbers* $-435 + 650 = 115$
$y = \frac{115}{5} = 23$		*Divide across by 5 to get our **y value***

To get our **x value** we sub in our y value into either Equation 1 or Equation 2

$x + y = 87$	**Equation 1**
$x + (23) = 87$	*Sub in y value*
$x = 87 – 23$	*x's to one side, numbers to the other*
$x = 54$	*the **x value***

$x = 54$ **AND** $y = 23$

Algebraic Long Division –

Follow the steps on the right hand side of the below examples for the procedure to use in Algebraic Division

Example – Divide x – 2 into $x^3+3x^2-4x-12$

x^2+5x+6	
$x-2\overline{)x^3+3x^2-4x-12}$	*Divide x^3 by x and **put the answer at the top,** x^2*
$\underline{x^3-2x^2}$	*Multiply (x –2) by x^2 and **put answer under first two terms***
$5x^2-4x-12$	*Subtract (change signs) and divide $5x^2$ by x*
$\underline{5x^2-10x}$	*Multiply (x –2) by 5x*
6x – 12	*Subtract (change signs) and divide 6x by x*
<u>6x – 12</u>	*Multiply (x –2) by 6*
0	*Subtract (change signs)*

(x^2+5x+6 **)** ***Answer***

We are basically repeating the same step 3 times. You will know that your answer is correct if when you subtract the last set of terms your answer is 0

If we are asked to divide into an expression that has some parts missing, for example there is no x^2 part, we leave space for any that may appear.

Example – Divide $27x^3-1$ by 3x -1

$9x^2+3x+1$	
$3x-1\overline{)27x^3\ \_\_\_\_\_\_\_\_-1}$	*Divide 27 x^3 by 3x and **put the answer at the top,** 9 x^2*
$\underline{27x^3-9x^2}$	*Multiply (3x – 1) by $9x^2$ and **put answer underneath***
$9x^2\ \_\_\_\_\_-1$	*Subtract (change signs) and divide $9x^2$ by 3x*
$\underline{9x^2-3x}$	*Multiply (3x – 1) by 3x*
3x – 1	*Subtract (change signs) and divide 3x by 3x*
<u>3x – 1</u>	*Multiply (3x – 1) by 1*
0	*Subtract (change signs)*

($9x^2+3x+1$ **)** ***Answer***

Problem Solving using Algebra –

This will be a written question which will ask you to write an expression using x to describe a situation. You will then be asked to alter this expression given new information.

Both the original and the new expressions are then used in some way to give us a quadratic equation, which we solve to find the values of x.

Example – In the first week of a club draw, x people shared equally in a prize of €400
(a) In terms of x how much was the value of each share?
(b) The following week, (x + 6) people shared equally in the prize of €400. In this second week, each share was €15 less than each share in the first week.
Write an equation to represent this information.
Solve the equation to find x.

Firstly lets use x to write expressions for (a) and (b) above.

Expression (a)

Value of each share = $\dfrac{PRIZE}{Number.of.People} = \dfrac{400}{x}$

Expression (b)

Value of each share = $\dfrac{PRIZE}{Number.of.People} = \dfrac{400}{x+6}$

To solve for x we ask what other information are we told?

Value of each share in week 1 = Value of each share in week 2 PLUS €15

$\dfrac{400}{x} = \dfrac{400}{x+6} + \dfrac{15}{1}$ *Write out equation above using our expressions*

$\dfrac{400(x+6)(1) = 400(x)(1) + 15(x)(x+6)}{x(x+6)(1)}$ *Find common denominator to remove fractions*

$400(x+6) = 400(x) + 15(x)(x+6)$ *Get rid of the bottom*

$400x + 2400 = 400x + 15(x^2 + 6x)$ *Simplify*

$400x + 2400 = 400x + 15x^2 + 90x$ *Simplify*

$-15x^2 + 400x - 90x - 400x + 2400 = 0$ *Take all to one side*

$-15x^2 - 90x + 2400 = 0$ *Simplify and take to one side*

$15x^2 + 90x - 2400 = 0$ *Change Signs*

$x^2 + 6x - 160 = 0$ *Divide across by 15 to simplify*

$(x+16)(x-10) = 0$ *Factorise*

$(x+16) = 0 \quad (x-10) = 0$ *Let each bracket = 0*

$x = -16 \;\; and \;\; x = 10$ *Values for x*

x = 10 *because x can't be a minus number (can't have minus number of people!)*

Example – A box of drinking chocolate powder costs €3.60 with x being the number of grams in the box.
(a) Write an expression in terms of x to represent the cost of 1 gram of the powder.
(b) During a promotion an extra 30 grams is added to the box which remains at a selling price of €3.60. Write an expression to represent the cost of one gram of the powder during the promotion.
Each gram of powder in this case costs 1c less.
Write an equation in x to represent the above.
Solve the equation for x.

Expression (a)

Cost of each gram = $\dfrac{Cost.Of.Box}{Grams.in.the.Box} = \dfrac{360}{x}$ *Change €3.60 into 360c*

Expression (b)

Cost of each gram = $\dfrac{Cost.Of.Box}{Grams.in.the.Box} = \dfrac{360}{x+30}$

To solve for x we ask what other information are we told?

Cost of 1 gram before promotion = Cost of 1 gram after promotion PLUS 1c

$\dfrac{360}{x} = \dfrac{360}{x+30} + \dfrac{1}{1}$ *Write out equation above using our expressions*

$\dfrac{360(x+30)(1) = 360(x)(1) + 1(x)(x+30)}{x(x+30)(1)}$ *Find common denominator to remove fractions*

$360(x+30) = 360(x) + (x)(x+30)$ *Get rid of the bottom*

$360x + 10800 = 360x + x^2 + 30x$ *Simplify*

$-x^2 + 360x + 10800 - 360x - 30x = 0$ *Simplify*

$-x^2 - 30x + 10800 = 0$ *Take all to one side*

$x^2 + 30x - 10800 = 0$ *Change Signs*

$(x+120)(x-90) = 0$ *Factorise*

$(x+120) = 0 \quad (x-90) = 0$ *Let each bracket = 0*

$x = -120 \text{ and } x = +90$ *Values for x*

x = 90 *because x can't be a minus number (can't have minus number of grams)*

Problem Solving involving Areas using Algebra –

The method used in these types of questions is as on the previous two pages but involves knowledge of the area formulae of various shapes.

Example – The length of a rectangle is 5cm more than its breadth. The area of the rectangle is $104cm^2$. Calculate the measurements of each side.

Area – Length x Breadth	*Write down formula for area*
Breadth = x	*Let x be the breadth of the shape*
Length = x + 5	*Let x + 5 be the length of the shape*
Area = $x(x+5)$	*Sub these into the area formula*
$= x^2 + 5x$	*This is the area of the shape*
$x^2 + 5x = 104$	*Let this equal the area of 104*
$x^2 + 5x - 104 = 0$	*Bring everything to the one side*
$(x+13)(x-8) = 0$	*Factorise the resultant quadratic*
$x = -13$ $\quad$ $x = 8$	*Let each bracket = 0 to solve*

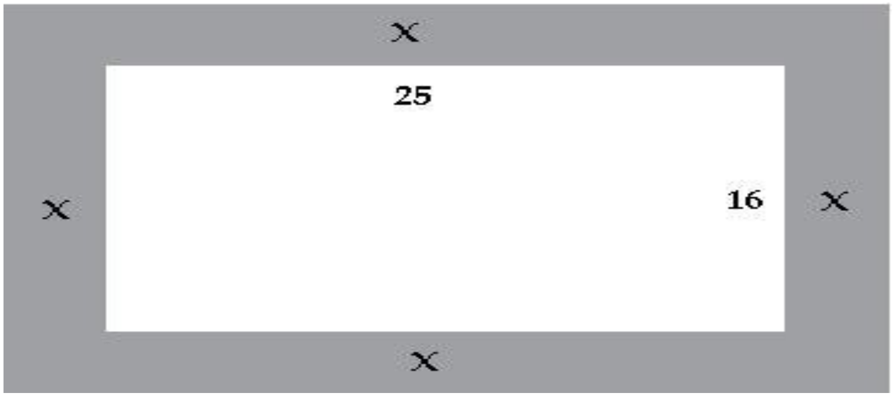
x = 8 only because measurements cannot be minus numbers

Breadth = 8cm

Length = 8 + 5 = 13cm

TURN THE PAGE FOR ANOTHER EXAMPLE

Example - A path of uniform width surrounds a rectangular lawn measuring 25m by 16m. The area of the path is 180. Find the width of the path.



Area of lawn = 400	*Multiply 25 by 16*
Area of path = 180	*We are given this*
Total area of garden = 580	*Lawn plus the path (400 + 180)*

Length of garden = x + 25 + x = 2x + 25
Breadth of garden = x + 16 + x = 2x + 16

Area of Garden = (2x + 25)(2x + 16)	*Area = Length by Breadth*
$2x(2x+16)+25(2x+16)=4x^2+32x+50x+400$	*Open brackets and multiply*
$=4x^2+82x+400$	*Area of Garden*
$4x^2+82x+400=580$	*Let area formula equal 580, above*
$4x^2+82x+400-580=0$	*Bring everything to one side*
$4x^2+82x+180=0$	*Simplify*
$2x^2+41x-180=0$	*Simplify by dividing across by 2*
$(2x+45)(2x-4)=0$	*Factorise to solve*
$2x+45=0 \qquad 2x-4=0$	*Let each bracket = 0*
$2x=-45 \qquad 2x=4$	*x's to one side, numbers to the other*
$x=\frac{-45}{2} \qquad x=\frac{4}{2}$	*Divide across by 2*
$x=-22.5 \qquad x=2$	***x values***

Because measurement (length) must be positive, the width of the path is :

x = 2m

Inequalities –

These are similar to simple equations but use the greater than, less than signs, greater than or equal to and less than or equal to signs (>, <, $\geq$ and $\leq$).
The rules for solving are similar to solving normal equations however one important difference is that if we decide to change all the signs we MUST change the direction of the inequality also.

Example if $-x \geq 4$ then $x \leq -4$ *Change the signs, change the direction of inequality*

If asked to draw a number line we must pay close attention to whether the numbers are:
$x \in R$ - these are rational numbers, fractions, decimals etc and are illustrated on the number line with a **shaded line.**
$x \in Z$ - these are integers, all positive and negative whole numbers and are illustrated on the number line with **dots.**
$x \in N$ - there are natural numbers, positive whole numbers and are illustrated on the number line with **dots.**

Example **Solve $5x + 1 \geq 4x + 3$ for $x \in R$ and illustrate on number line.**

$5x + 1 \geq 4x + 3$

$\longrightarrow$ $5x - 4x \geq 3 - 1$ *x's to one side, numbers to the other*

$\longrightarrow$ $x \geq 2$ *Evaluate*

To show this on a number line

-3 -2 -1 0 1 2 3 4 $x \in R$

If the question had stated that $x \in N$ or $x \in Z$ we would use the following number line

-3 -2 -1 0 1 2 3 4 $x \in N$, $x \in Z$

Example – **A** is the set $3x - 2 \leq 4$ $x \in Z$

B is the set $\frac{1-3x}{2} \leq 5$ $x \in Z$

A	**B**
$3x \leq 4 + 2$	$\frac{1-3x}{2} \leq 5$
$3x \leq 6$	$1 - 3x \leq 10$
$x \leq 2$	$-9 \leq 3x$
	$-3 \leq x$

A $\cap$ B would be what is common to A and B
In this case the numbers –3,-2,-1, 0, 1, 2

If asked to illustrate on the number line the set A $\cap$ B:

-4 -3 -2 -1 0 1 2 3 4 $x \in Z$

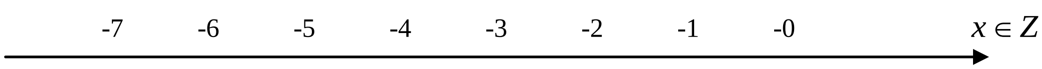
Example – $2x-1 \le x-2 \le 3x+10$

In this example we spilt into two inequalities both having the middle term x – 2

$2x-1 \le x-2$	**AND**	$x-2 \le 3x+10$
$2x-x \le -2+1$		$-10-2 \le 3x-x$
$x \le -1$		$-12 \le 2x$
		$\frac{-12}{2} \le x$
		$-6 \le x$

So x is anything less than or equal to –1 and x is everything greater than or equal to –6
We write this like $-6 \le x \le -1$

If asked to illustrate on the number line:



Rearrange –

This involves using our algebra skills to rearrange equations
Step 1 Remove brackets or fractions if necessary
Step 2 Take anything with the letter we are looking for to the left of the '=' and everything else to the right of the '='.
Step 3 If there is more than one term now on the left, factorise to get the letter alone.
Step 4 Divide across by the term next to the letter we want to isolate.

Example – Express x in terms of a,b and c (this means get x by itself on left of the '=')

$$ax + b = c$$

$ax + b = c$	
$a\mathbf{x} = c - b$	*Bring everything with an **x** to the left, everything else to the right.*
$x = \dfrac{c-b}{a}$	*Divide across by a to isolate **x***

Example – Express b in terms of a and c (this means get b by itself on left of the '=')

$$\frac{8a-5b}{b} = c$$

$8a - 5\mathbf{b} = \mathbf{b}c$	*Get rid of fraction by multiplying across by **b***
$-5\mathbf{b} - \mathbf{b}c = -8a$	*Bring everything with a **b** to the left, everything else*
$5\mathbf{b} + \mathbf{b}c = 8a$	*to the right.* *Change all the signs.*
$\mathbf{b}(5 + c) = 8a$	*Factorise by taking out **b***
$b = \dfrac{8a}{(5+c)}$	*Divide across by (5 + c) to isolate **b***

Example – Express t in terms of p and q (this means get t by itself on left of the '=')

$$p = \frac{q-t}{3t}$$

$3\mathbf{t}(p) = q - \mathbf{t}$	*Get rid of fraction by* multiplying across by **3t**
$3\mathbf{t}p + \mathbf{t} = q$	*Bring everything with a **t** to the left, everything else to the right.*
$\mathbf{t}(3p + 1) = q$	*Factorise by taking out **t***
$t = \dfrac{q}{(3p+1)}$	*Divide across by (3p + 1) to isolate **t***

Functions –

Questions with functions involve replacing the x in an expression with a number.

Example – $f(x) = 3x + 5$ **Calculate** $f(3)$ **and** $f(-1)$

$f(x) = 3x + 5$	*Write out the original function*
$f(3) = 3(3) + 5 = 9 + 5 = \mathbf{14}$	*To get* $f(3)$ *replace the x with a 3*
$f(-1) = 3(-1) + 5 = -3 + 5 = \mathbf{2}$	*To get* $f(-1)$ *replace the x with a –1*
$f(3) = 14$ and $f(-1) = 2$	***Answers***

The numbers we put in to the function are called the **domain.**
The numbers we get out are called the **range.**

Example – f:x $\rightarrow$ ax + b

This line cuts the x axis at (3,0) and the y axis at (0, -2). Calculate **a** and **b**.

(3, 0) $\longrightarrow$ the y part is 0 when x is 3

let f:x = 0	
ax + b = 0 when x = 3	*because f:x means y*
a(3) + b = 0	*sub in x =3*
3a + b = 0	***Equation 1***

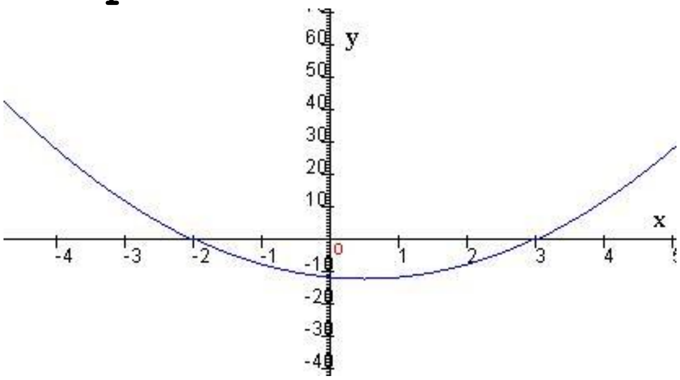
(0, -2) $\longrightarrow$ the x part is 0 when the y part is –2

let f:x = -2	
ax + b = -2 when x = 0	*because f:x means y*
a(0) + b = -2	*sub in x = 0*
b = -2	***Equation 2 (b value)***

3a + b = 0	*Write out equation 1 again*
3a + (-2) = 0	*Sub in above value for b*
3a –2 = 0	*Simplify*
3a = 2	*x's to one side, numbers to the other*
$a = \frac{2}{3}$	*Divide across by 3*

In the above example Equation 2 gave us the b value.
If though equation 1 and 2 both have a's AND b's we must do a simultaneous equation.

Example -



Left we have a graph of the function

$f: x \to 2x^2 + ax + b$

(i)Evaluate **a** and **b.**

(ii) If the graph also contains (k,3k) evaluate k if k > 0

We can see that the curve crosses the x axis at (–2,0) and (3,0)
So x = -2 and x = 3 when y = 0

$f: x \to 2x^2 + ax + b$

(i) $2x^2 + ax + b = 0$ when x = 3 and when x = -2 ***Remember f:x = y***

For (-2, 0)
Sub in x = -2 and let f:x = 0

$2(-2)^2 + a(-2) + b = 0$

$2(4) + -2a + b = 0$

$8 - 2a + b = 0$

$-2a + b = -8$

$2a - b = 8$

EQUATION 1

For (3, 0)
Sub in x = 3 and let f:x = 0

$2(3)^2 + a(3) + b = 0$

$2(3)^2 + a(3) + b = 0$

$2(9) + 3a + b = 0$

$18 + 3a + b = 0$

$3a + b = -18$

EQUATION 2

Use the above equations to form a simultaneous equation

$2a - b = 8$ **EQUATION 1**

$3a + b = -18$ **EQUATION 2**

5a = -10
a = -2

Sub this into Equation 1 or 2 and get b = - 12

$f: x \to 2x^2 + ax + b$

$f: x \to 2x^2 - 2x - 12$

(ii) If the point (k, 3k) is on the curve then when x = k, f:x = 3k. ***Remember f:x = y***

$f: x \to 2x^2 - 2x - 12$

$2x^2 - 2x - 12 = 3k$ *when x = k*

$2(k)^2 - 2k - 12 = 3k$ *sub in x = k*

$2k^2 - 2k - 12 - 3k = 0$ *Bring everything to the left hand side*

$2k^2 - 5k - 12 = 0$ *Simplify*

Using the –b formula we get k = -1.5 and k = 4
k > 0 therefore **k = 4**

Page 5 for revision of the –b formula

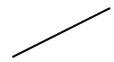
Graphs –

Remember that to put a point on a graph we need an x part and a y part (x, y).
To get the points we put numbers into a function. The numbers we put in are the x values, the numbers we get out are the y values.

Very Important f:x or f(x) is another way or saying y.

You will normally be given the x values to put in (known as the domain) such as $-3 \le x \le 2$ which means put in all the numbers between –2 and 3.

There are a number of different graphs we can be asked to draw.

1. A linear function like $f(x) = 3x + 5$ will give you a **straight line.** 

2. A quadratic function such as $f(x) = 3x^2 + 5x + 4$ will give you a **curve with only one turning point** (that is one place on the curve where it changes direction). It will be  shaped or  shaped.

1. To easiest way to draw a line if you are not given the domain (i.e told what points to put in) is to work out the points where the line cuts the x and y axis.
A line cuts the x axis at y = 0
A line cuts the y axis at x = 0

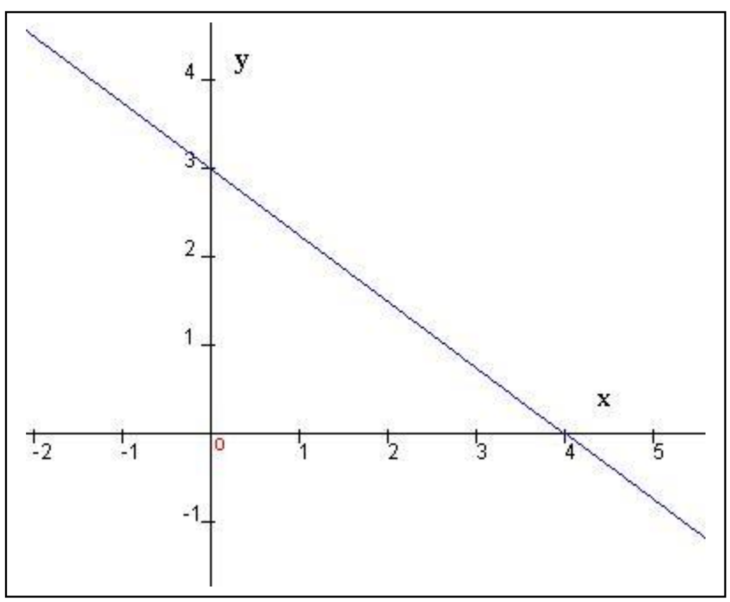
Example – Draw the line 3x + 4y = 12

Cuts the x axis at y = 0
3x + 4y = 12
3x + 4(0) =12
3x = 12
x = 4

(4, 0)

Cuts the y axis at x = 0
3x + 4y = 12
3(0)+ 4y =12
4y =12
y = 3

(0, 3)



TURN OVER FOR AN EXAMPLE OF A CURVE AND LINE ON THE SAME GRAPH

2. If you are given the domain the graph is drawn by putting the x values into the function and getting a corresponding y value.

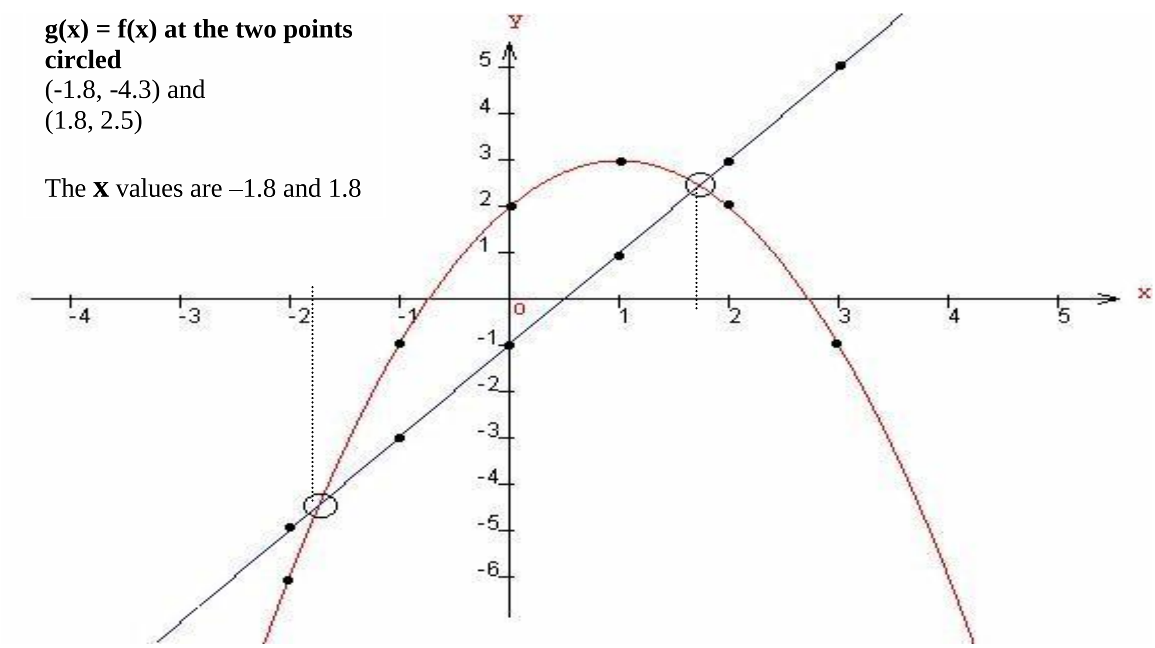
Example - $f(x) = 2 + 2x - x^2$ Draw a graph in the domain $-2 \le x \le 3$

1. Using the same axis and scale draw a graph of the line $g(x) = 2x - 1$ in the domain $-2 \le x \le 3$
2. Find the values of x for which g(x) = f(x)

$f(x) = 2 + 2x - x^2$	**Points for graph**
$f(-2) = 2 + 2(-2) - (-2)^2 = 2 - 4 - 4 =$ **-6**	**(-2, -6)**
$f(-1) = 2 + 2(-1) - (-1)^2 = 2 - 2 - 1 =$ **-1**	**(-1,-1)**
$f(0) = 2 + 2(0) - (0)^2 = 2 - 0 - 0 =$ **2**	**(0, 1)**
$f(1) = 2 + 2(1) - (1)^2 = 2 + 2 - 1 =$ **3**	**(1, 0)**
$f(2) = 2 + 2(2) - (2)^2 = 2 + 4 - 4 =$ **2**	**(2, 2)**
$f(3) = 2 + 2(3) - (3)^2 = 2 + 6 - 9 =$ **-1**	**(3,-1)**

$g(x) = 2x - 1$	**Points for graph**
$g(-2) = 2(-2) - 1 = -4 - 1 =$ **-5**	**(-2,-5)**
$g(-1) = 2(-1) - 1 = -2 - 1 =$ **-3**	**(-1,-3)**
$g(0) = 2(0) - 1 = 0 - 1 =$ **-1**	**(0,-1)**
$g(1) = 2(1) - 1 = 2 - 1 =$ **1**	**(1, 1)**
$g(2) = 2(2) - 1 = 4 - 1 =$ **3**	**(2, 3)**
$g(3) = 2(3) - 1 = 6 - 1 =$ **5**	**(3, 5)**

g(x)



f(x)

Reading Graphs –

Once the graph has been drawn you can be asked a number of questions about it. Below are examples of the most common types. Make sure you understand each one.

What are the negative areas and positive areas of the curve?
Negative areas are parts of a graph below the x axis.
Can be written in exam as; Give the range of values of x for which $f(x) < 0$
Positive areas are parts of a graph above the x axis.
Can be written in exam as; Give the range of values of x for which $f(x) > 0$
Give the x values, not the points.

What are the areas where the curve is increasing and decreasing?
Read the graph from left to right, the curve is increasing when it is going up and decreasing when it is going down (think of a roller coaster).
Give the x values, not the points.

What are the roots of the equation f(x) = 0?
Here they are asking what are the x values when y = 0 (remember f(x) = y).
This means what are the x values where the curve cuts the x axis.
Give the x values.

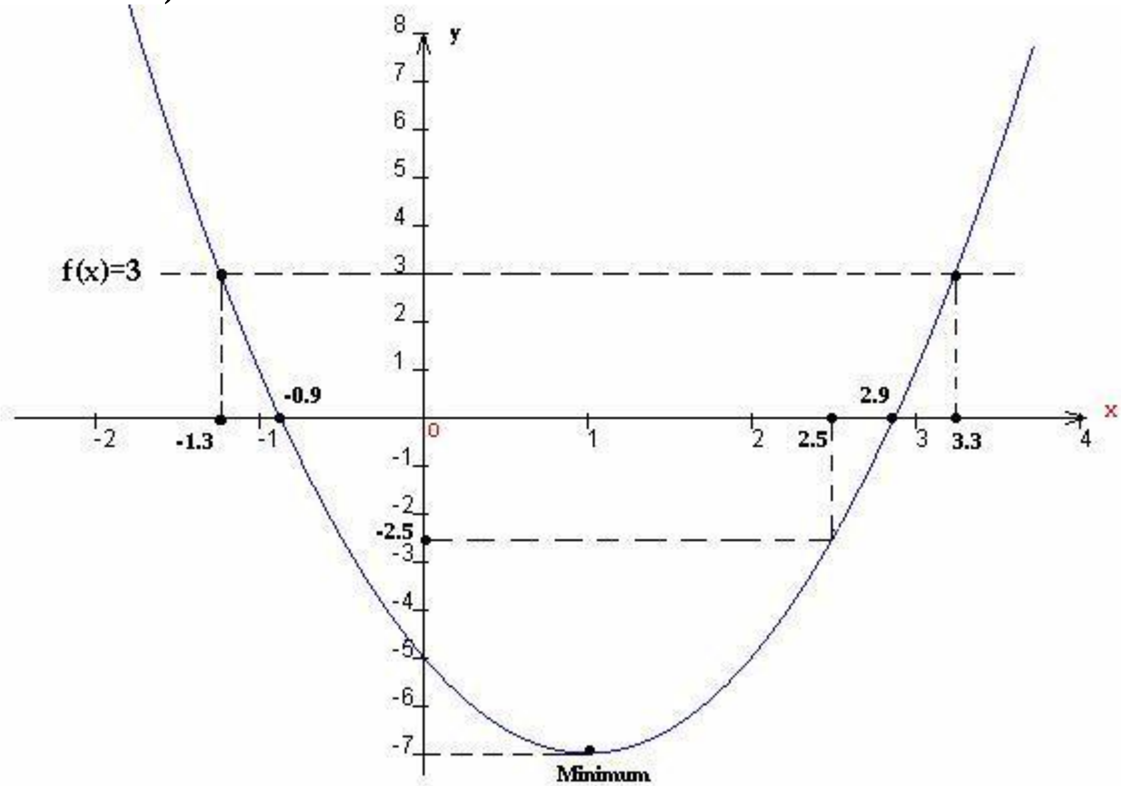
What are the roots of the equation f(x) = -3?
Here they are asking what are the x values when y = -3. To do this draw a horizontal line through y = -3. Where this line cuts your curve read off the x values.
Give the x values.

What are the values of f(4) and f(-2).
This means what are the y values when x = 4 and when x = -2. Draw vertical lines through x = 4 and x = -2 and where these lines cut our curve read off the y values.
Give the y values this time.

What are the maximum and minimum points?
These are the highest and lowest points of turning on your curve.
Give the points in the form (x, y) unless they ask specifically for x or y values.

TURN OVER FOR AN EXAMPLE THAT INCLUDES ALL OF THESE

Below is a graph of the function $f : x \rightarrow 2x^2 - 4x - 5$ (in the exam you'll have to draw it first).



Use the graph to find:

1. **f(2.5)**

Go to where x = 0.5 and draw dotted line down to where it cuts the curve.
y = -2.5

2. **The minimum point on the graph**

Go to lowest point on the curve. (1, -7)

3. **The range of values of x for f:x > 0 (where is graph positive?)**

Graph is positive above x axis so before x = -0.9 and after 2.9
We write this $x < -0.9$ and $x > 2.9$

4. **The range of values for x for which f:x < 0 (where is graph negative?)**

Graph is negative below x axis so after -0.9 and before 2.9
We write this $-0.9 < x < 2.9$

5. **The range of values for f:x = 3**

f:x is the same as saying y so draw a dotted line through y = 3
Where it cuts the curve read down the x values x = -1.3 and x = 3.3

6. **The range of values for f:x is increasing**

Curve is increasing from x = 1 all the way to infinity
We write this x > 1

7. **The range of values for which f:x is decreasing**

Curve decreasing from minus infinity to x = 1
We write this x < 1

8. **The values of x for which $2x^2 - 4x - 5 = 0$**

Similar to 5 above. $2x^2 - 4x - 5 = 0$ is the same as saying f:x = 0 or y = 0
y = 0 where the curve cuts the x axis, x = -0.9 and x = 2.9

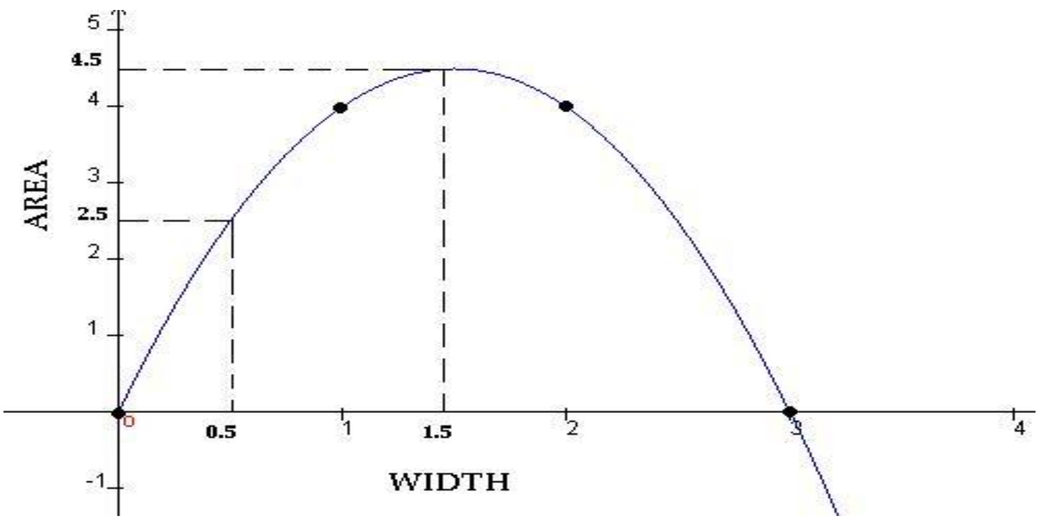
Example – The area of a rectangle is given by the function $f : x \to 6x - 2x^2$ where x stands for the width in meters.
Draw the graph for $0 \leq x \leq 3$

Using the graph estimate

1. The area of the rectangle when the width, x, is 0.5m
2. The maximum possible area of the rectangle
3. The width of the rectangle at this area
4. Two possible values of the width when the area is $4m^2$

Putting the points in we get.

$f : x \to 6x - 2x^2$	**Points for graph**
$f : 0 \to 6(0) - 2(0)^2 = 0$	**(0, 0)**
$f : 1 \to 6(1) - 2(1)^2 = 6 - 2 = 4$	**(1, 4)**
$f : 2 \to 6(2) - 2(2)^2 = 12 - 8 = 4$	**(2, 4)**
$f : 3 \to 6(3) - 2(3)^2 = 18 - 18 = 0$	**(3, 0)**



1. The area of the rectangle when the width, x, is 0.5m
Go to x = 0.5 and draw dotted line to where it meets curve, read across the y value.
When width is 0.5 the Area is $2.5m^2$
2. The maximum possible area of the rectangle
Go to the maximum (highest) point of the curve, draw a dotted line across to read area.
At the highest point on the curve the area is $4.5m^2$
3. The width of the rectangle at this area
Bring the maximum point down and read the width value.
At the max point the width is 1.5m
4. Two possible values of the width when the area is $4m^2$
Read off the x values for which the y value (the area value) is equal to 4.
In this case for x = 1 and x = 2